No. 91-860

In the Supreme Court of the United States.

October Term, 1991

UNITED STATES DEPARTMENT
OF COMMERCE, et al.,
Appellants

v.

STATE OF MONTANA, et al.,
Appellees

On Appeal From the United
States District Court
for the District of Montana

LODGING OF THE COMMONWEALTH OF MASSACHUSETTS AS AMICUS CURIAE

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EDITOR'S NOTE

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AFFIDAVIT OF H. PEYTON YOUNG

I. BACKGROUND

1. I am Professor of Public Policy and Applied Mathematics at the University of Maryland. My curriculum vitae has been lodged with the Court. I have spent over fifteen years studying the mathematical theory and the legislative history of apportionment in the United States. The results of these studies are given in my book with Michel L. Balinski entitled *Fair Representation: Meeting the Ideal of One Man One Vote*, Yale University Press, 1982 (hereinafter *Fair Representation*).

2. I submit this affidavit in support of the brief of the Commonwealth of Massachusetts before this Court as amicus curiae. References below to the "Stipulation" refer to the Stipulation of Facts entered into in the Massachusetts litigation and lodged with the Court.

II. THE NEAR AS MAY BE STANDARD

3. The history of apportionment legislation in the United States can be understood as a search for a formula that meets the constitutional ideal of one person, one vote as near as may be. The "near as may be" or close as practicable standard was first formulated by Daniel Webster in an address to the Senate in 1832: "The Constitution, therefore, must be understood, not as enjoining an absolute relative equality, because that would be demanding an impossibility, but as requiring of Congress to make the apportionment of representatives among the several States according to their respective numbers as near as may be. That which cannot be done perfectly must be done in a manner as near perfection as can be . . " *The Writings and Speeches of Daniel Webster, National Edition*. Boston: Little Brown, 1903, vol. 6, at 107-109.

4. Webster's address was provoked by an apportionment using the then-prevailing method of Jefferson. By the census of 1830, Jefferson's method gave 40 seats to New York (with a quota of 38.593) and 28 seats to Pennsylvania (with a quota of 27.117), but only 5 seats to Vermont (with a quota of 5.646) and only 1 seat to Delaware (with a quota of 1.517). Webster pointed out that by taking one seat from New York and giving it to Vermont, and

by taking one seat from Pennsylvania and giving it to Delaware, the representation of *all four* states would be brought closer to their quotas, that is, to their ideal numbers of seats. Hence the Jefferson apportionment did not meet the "near as may be" standard and, after 1840, was abandoned.

5. An apportionment meets Webster's "near as may be" standard if it is not possible to bring any state closer to its quota without moving some other state further away from its quota. (The quota of a state is the ideal share of a representative for each individual times the number of the individuals in that state.) An apportionment with this property is also said to be "near the quota." *Fair Representation* at 132.

6. The most natural way to find an apportionment that is "near the quota" would appear to be the following: round up every state with fractional remainder above .5, and round down every state with fractional remainder below .5. But this rounding method may apportion too many or too few seats. For example, according to the 1990 census there are 29 states with fractional remainders above one-half. If all of them were rounded up, then 438 seats would be apportioned instead of the required 435. Hence at least three states with quotas greater than one-half must be rounded down. The problem is to determine which ones they should be.

7. Webster's solution to this problem was the following: "Let the rule be that the population of each state be divided by a common divisor, and, in addition to the number of members resulting from such division, a member shall be allowed to each State whose fraction exceeds a moiety of the divisor." *The Writings and Speeches of Daniel Webster* at 120. Thus, the rounding threshold for Webster is always .5, and a divisor is found that produces (presently) 435 seats. See also Stipulation 73.

8. Webster's method always yields apportionments that meet the "near as may be" standard. In other words, if some states can be brought closer to their quotas without moving any other states further from their quotas, the Webster method will do so. Moreover, Webster apportionments never produce a situation in which one state with fraction above .5 is rounded down and simultaneously another state with fraction below .5 is rounded up. *Fair Representation* at 132. Such a situation is called a "reversal." A reversal does not meet the near as may be standard, because rounding the other way -- that is, rounding up the state with fraction above .5 and rounding down the state with fraction

below .5 -- brings both states closer to their quotas, and brings the average share of a representative in both states closer to the national average share of a representative.

9. By contrast, the current statutory method of Hill (as well as the methods of Dean, Adams, and Jefferson) often produce reversals, as I shall show below. Hence they do not meet Webster's near as may be standard. In fact, recognized methods of apportionment (e.g., Adams and Jefferson) often stray far from the quotas. For example, if Adams' method were applied to the 1990 census, it would allot California (quota 52.124) only 50 seats. Jefferson's method is similarly skewed the other way: in 1990 it would have given California 54 seats. These two methods are "rationally" tied to the states' populations, but in operation they often give results that do not come anywhere near the ideal of equal representation.

III. THE PAIRWISE COMPARISONS APPROACH

10. Webster's method is based on the premise that every state should be as close as possible to its ideal number of seats, that is, its quota. A different approach was first suggested in 1911 by Joseph Hill, a statistician at the Census Bureau. Hill's idea was to apportion seats among the states so that the relative difference in representation between every two states is as small as possible. This idea was later refined and elaborated by a Harvard mathematician named Edward V. Huntington.

11. Huntington observed that, in a perfect apportionment, the number of persons per representative --the average district size--would be the same in every state. Likewise, the the number of representatives per person--the average share of a representative -- would be the same in every state. In practice, there will almost always be differences between the states because of the rounding problem.

12. An apportionment favors one state relative to another if each resident of the first state has a larger average share of a representative than each resident of the second state.

13. Huntington argued that the goal of apportionment should be to minimize the inequality in representation between every two states. In other words, it should not be possible to transfer a seat from the more favored state to the less favored state and reduce the amount of inequality between them. This is known as the "pairwise comparisons" approach to

apportionment. To make this approach workable, it is necessary to have a numerical measure of the "amount of inequality" between two states.

14. A natural measure of inequality between states is the absolute difference in their residents' average share of a representative, that is, the larger share minus the smaller share. Huntington showed that Webster's method is the only one that minimizes this measure of inequality.

15. A second measure of inequality between states is the absolute difference in the states' average district sizes, that is, the larger district size minus the smaller district size. Huntington showed that Dean's method is the only one that minimizes this criterion of inequality.

16. A third measure of inequality between states is the relative difference in the average share of a representative in each state. (The relative difference between the shares is the absolute difference between them divided by the smaller of the two shares.) This notion of inequality is minimized by Hill's method. Hill's method also minimizes the relative difference between the average district sizes of every two states.

17. Huntington showed that two less natural measures of inequality lead to the methods of Jefferson and John Quincy Adams. He showed further that the five methods -- Adams, Dean, Hill, Webster, and Jefferson -- are the only ones that result from minimizing one of his measures of inequality.

18. Huntington maintained that Hill's method (which he dubbed the method of "equal proportions" and which he attributed to himself rather than Hill) is the most appropriate of these five methods. One of his principal arguments was that Hill's method minimizes the relative difference in the average share of a representative on a pairwise basis, and also the relative difference in average district size on a pairwise basis. Hence it is not necessary to choose between average share of a representative and average district size as the measure of one person one vote, since they lead to the same result. This argument is premised, however, on the assumption that the *relative* difference between states is a more appropriate statistical measure of inequality than the *absolute* difference. As demonstrated below, this premise is faulty if the goal is equal representation for each person.

IV. SHARE OF A REPRESENTATIVE AS THE PROPER MEASURE OF ONE PERSON ONE VOTE

19. I understand one person one vote to mean that every person should have equal representation in the House. Within each state, equality can be achieved by creating districts with equal populations. In this case there is no discrepancy between average district size and average share of a representative, for if all districts have the same number of persons, then *a fortiori* all persons have an equal share of a representative. In the interstate context, however, where equal-sized districts are a practical impossibility, the two concepts are not equivalent.

20. To illustrate, suppose that a state has a population of 750,000 and that the national average district size is 500,000. The state is entitled to 1.5 seats (its quota). If it gets 2 seats then its average district size is 375,000. If it gets 1 seat its average district size is 750,000. It must get a whole number of seats, so it must either be advantaged or disadvantaged in any particular apportionment. Over several apportionments, however, the advantages and disadvantages should balance out.

21. For example, suppose that the population of the state remains fixed at 750,000 over two successive censuses, and that the national average district size remains fixed at 500,000. Thus the national average share of a representative is 1 for every 500,000 persons. If the state gets 1 seat in the first census and 2 seats in the second census (or the other way around) it has been treated fairly on average. This can be verified in two ways. First, the average number of seats that the state received over the two censuses is 1.5, which is the number it should receive (its quota). A second way of verifying fair treatment is to compute the average share of a representative in each of the two censuses. In the first census the average share was 1 divided by 750,000 or 1.33 representatives per million persons. In the second census it was 2 divided by 750,000 or 2.66 representatives per million persons. The average share over the two censuses was therefore 2 representatives per million persons, that is, one for every 500,000, which is exactly what it should be.

22. A different conclusion emerges if one compares average district sizes, however. The average district size in the first census was 750,000 and in the second census it was 375,000. Over the two censuses the average district size was therefore 562,500, which is larger than the national average (500,000). This would lead to the erroneous conclusion that the persons in the state had been underrepresented on average over the two censuses.

The fact is that the *districts* in the state were larger (on average over the two censuses) than the national average district, but the *persons* were treated, on average, equally with everyone else.

23. This example shows why the proper measure of one person one vote is the individual's share of a representative and not the average district size. If exact equality can be achieved (as in the intrastate context) then the two concepts are equivalent. They are not equivalent when, as in the interstate context, there are wide discrepancies in district size.

24. The effect of using average district size to measure inequality is to bias the outcome in favor of small states. To illustrate, consider an actual situation that arose in 1920. New York had a population of 10,380,589 and New Mexico had a population of 353,428. Nationally, the ideal share of a representative was 4.135 per million. Thus New York's quota (the number of persons in New York times the ideal share) was 42.919, whereas New Mexico's quota was 1.461. The natural solution would be to give 43 seats to New York and 1 seat to New Mexico (which is what Webster's method would do).

25. The average-district-size test, however, yields a different result. The average district size in New York (with 43 seats) would be 241,409, while the average district size in New Mexico (with 1 seat) would be 353,428. According to the average-district-size test, the "amount of inequality" between them is 112,019 persons per district. Suppose instead that New York received 42 seats and New Mexico received 2 seats. Then the average district sizes would be 247,157 in New York and 176,714 in New Mexico. The "amount of inequality" would then be 70,443 persons per district, which is smaller. By this reasoning, New York (with a quota of 42.92) would be rounded *down* to 42 seats and New Mexico (with a quota of 1.46) would be rounded *up* to 2 seats. This is a reversal. (See Par. 8.)

26. A reversal does not meet the near as may be standard. New York would be closer to its quota if it got 43 seats instead of 42, and New Mexico would be closer to its quota if it got 1 seat instead of 2. Similarly, New Yorkers' average share of a representative would be closer to the national average share if they got 43 seats instead of 42, and New Mexicans' average share of a representative would be closer to the national average share if they got 1 seat instead of 2. Both states would be closer to the ideal of one person one vote if the reversal were undone.

27. The methods of Adams and Hill also produce a reversal in this case. To see why Hill's method yields this result, suppose there were no reversal, so that New York receives 43 seats and New Mexico receives 1 seat. The average share of a representative would then be 4.142 per million in New York and 2.829 in New Mexico. According to the Hill relative difference criterion, the "amount of inequality" between the two states is the percentage by which 4.142 exceeds 2.829, which is about 46%. If instead New York receives 42 seats and New Mexico receives 2 seats, their average shares of a representative are 4.046 per million and 5.659 per million respectively. The relative difference between them is now about 40%, which is smaller than before. So according to Hill's method the reversal should stand.

28. The reason for this anomaly is Hill's reliance on the relative difference test, which is not appropriate here (see Par.33-45). Webster's method uses the absolute difference between shares of a representative as the criterion of inequality, which does not produce reversals. For example, in 1920, Webster's method would have allotted New York 43 seats and New Mexico 1 seat. The absolute difference between their shares of a representative would have been 1.313 per million. If instead New York received 42 seats and New Mexico received 2 seats (as it would under Hill), the absolute difference in their shares of a representative would have been 1.613 per million, which is larger. Therefore it makes more sense to give New York 43 seats and New Mexico 1 seat, and this involves no reversal.

29. This is not the only reversal that Hill's method would have produced in 1920 had it been the statutory method. Virginia (quota 9.547) would have been rounded down to 9 seats while Rhode Island (quota 2.499) would have been rounded up to 3 seats. North Carolina (quota 10.581) would have been rounded down to 10 seats while Vermont (quota 1.457) would have been rounded up to 2 seats.

30. By contrast, in 1920, Webster's method would have rounded *up* all states with quotas above one-half and rounded *down* all states with quotas below one-half. There would have been no reversals. This is because Webster's method minimizes the absolute, instead of the relative difference, in the shares of a representative as between every two states.

31. Hill's method has actually produced a number of reversals since it was adopted in 1941. In 1970, for example, Hill's method rounded up South Dakota (quota 1.435) to 2 seats, while it rounded down Connecticut (quota 6.503) to 6 seats. It also produced a

reversal in 1960. In every one of these cases, the Webster solution would have produced no reversals, and brought all of the affected states closer to their quotas.

32. It is also significant that in every case where Hill's method produces a reversal, the less populous state is the one that is favored (rounded up) and the more populous state is the one that is disfavored (rounded down). This is part of a systematic pattern of bias in which Hill's method tends to give more representation to residents of small states than to residents of large states, as I shall explain below (Par. 46-63).

V. RELATIVE DIFFERENCE VERSUS ABSOLUTE AND PERCENTAGE DIFFERENCE

33. The Hill method is based on the premise that the relative difference is the right way to measure the difference between two numbers. When the goal is to minimize deviation from the ideal of equality, however, the relative difference is not the right way.

34. The *absolute difference* between two numbers is the larger number minus the smaller number. The *relative difference* between two numbers is the absolute difference between them divided by the smaller of the two numbers.

35. When two numbers are being considered in isolation, and there is no common standard or benchmark to which they may be compared, both the absolute and relative differences are appropriate ways to measure "difference." For example, if per capita income in one country is $4000 and per capita income in another country is $6000, then the absolute difference is $2000 and the relative difference is 50%.

36. When there is an average or ideal value to which the numbers can be compared (as in apportionment), the relative difference is not appropriate. To illustrate, suppose that the average per capita income in a certain country is $6000 and that one region of the country has a per capita income of $4000, while another region of the same country has a per capita income of $8000. In the first region per capita income is 33 1/3% below the national average and in the second region it is 33 1/3% above the national average. But the relative difference between the first region's income and the national average income is 50% ($6000 is 50% larger than $4000), while the relative difference between the second region's income and the national average income is 33 1/3% ($8000 is 33 1/3% larger than $6000). The relative difference suggests that the deviation below the norm in the first

region is larger than the deviation above the norm in the second region, which is not the case.

37. When measuring differences from an ideal, either the absolute difference or the percentage difference from the ideal should be employed. The relative difference is neither standard nor appropriate as a statistical measure of deviation from a norm.

38. In apportionment there is a norm or ideal to which individuals can be compared, namely, the national average share of a representative. To measure the overall deviation from this ideal, a standard approach would be to compute the absolute difference between each individual's share and the national average share, square the result, and then sum the squared differences over all individuals. (This is the "variance" of the shares.) Webster apportionments minimize this overall measure of inequality, and they are the only apportionments that do so.

39. An alternative, but fallacious approach, would be to minimize the overall inequality ("variance") in district sizes. One would compute the absolute difference between each district size and the national average district size, square the result, and then sum over all 435 districts. The only apportionments that minimize this measure of inequality are Dean apportionments.

40. As already addressed, however, the district size is not the relevant standard for one person one vote. Each district corresponds to one representative. To bring all district sizes as near equality as possible is to treat all representatives as equally as possible. The relevant principle is to treat all citizens as equally as possible. The average share of a representative is the only appropriate standard for this purpose.

41. The Hill method's reliance on the relative difference between every two states results in apportionments that do *not* treat the citizens in these states as equally as possible. In 1990, Hill's method gives Massachusetts 10 seats, and its residents' share of a representative is 1.6586 per million persons (10 divided by 6,029,051). Oklahoma gets 6 seats, and its residents' share of a representative is 1.9002 per million persons (6 divided by 3,157,604). The difference between the shares is .2416. If Massachusetts were given 11 seats and Oklahoma 5 seats (as they would be under Webster's method) then the shares would be 1.8245 in Massachusetts and 1.5835 in Oklahoma. The difference is .2410,

which is smaller than under the current apportionment. Webster's method comes closer than Hill's to giving each person equal representation in the House.

42. Hill's method does not meet the near as may be test in percentage terms either. Together, Massachusetts and Oklahoma have 9,186,655 residents and they receive 16 seats under Hill's method. So the average share of a representative for the two states combined is 1.7417 per million. As noted in the preceding paragraph, the Hill apportionment results in a share of a representative in Massachusetts of 1.6586 per million, and a share of a representative in Oklahoma of 1.9002 per million. Thus Massachusetts is 4.77% below the average share for the two states combined and Oklahoma is 9.1% above the average share for the two states combined. Under the Webster apportionment the share in Massachusetts would be 1.8245 per million and the share in Oklahoma would be 1.5835. The former is 4.75% above the average share for the two states combined and the latter is 9.08% below the average share of the two states combined. Hence the Webster apportionment brings *both* states closer (in absolute *and* percentage terms) to the average share of a representative for the two states combined.

43. The state of Montana alleges that Dean's method comes closer than either Hill's or Webster's method to meeting the one person one vote standard. This is not the case. In 1990 Montana had a population of 803,655. Dean's method would allot it 2 seats, so its share of a representative would be 2.4886 per million. The state of Washington had a population of 4,887,941 and Dean's method would allot it 8 seats, so its average share of a representative would be 1.6367 per million. Thus the absolute difference between the shares is .8519. If instead Washington were allotted 9 seats and Montana were allotted 1 seat (as would be the case under either Webster's method or Hill's method), Washington's average share would be 1.8413 and Montana's would be 1.2443. The absolute difference under this apportionment is .597, which is smaller. Under Webster's method the individual residents of Washington and Montana have more equal representation than under Dean's method.

44. In summary, Webster's method comes as close as practicable to equalizing each person's share of a representative and to bringing all the states as close as practicable to their quotas.

45. By contrast, Hill's method fails to do either because it is based on the relative difference instead of absolute or percentage difference. The relative difference is not

appropriate when there is an ideal standard against which comparisons can be made, as in this case. Hill's method also leads to reversals: a state with fraction above .5 may be rounded down, while a state with fraction below .5 may be rounded up. Reversals can also occur under Dean's and Adams' methods. They never occur under Webster's method. Moreover, whenever reversals occur under the methods of Adams, Dean, or Hill, the state that is rounded down (the disfavored state) is always the larger of the two, while the state that is rounded up (the favored state) is always the smaller of the two. (By "larger" state I mean more populous state and by "smaller" state I mean less populous state.) In this sense these methods discriminate against the larger states.

VI. BIAS

46. Huntington asserted, nevertheless, that Hill's method is even-handed in its treatment of small and large states. He reached this conclusion by the following fallacious argument. Of the five methods that minimize pairwise measures of inequality (Adams, Dean, Hill, Webster, Jefferson), Adams favors small states the most and Jefferson's method favors large states the most. (In fact, Congress had rejected Jefferson's method in 1840 in part because it was biased in favor of large states. *Fair Representation* at 35.) Of the remaining three methods, Dean's favors the small states more than Hill's, and Hill's favors the small states more than Webster's. *Fair Representation* at 118. Therefore, said Huntington, Hill's method "has no bias in favor of either the larger or the smaller states." Edward V. Huntington, "The Apportionment Situation in Congress," *Science*, December 14, 1928, at 580.

47. This argument does not establish in any way that Hill's method is unbiased. It merely says that, of the five methods, Hill's favors small states less than two methods and favors small states more than two other methods. See Stipulation 69.

48. Huntington's views were challenged by the prominent statistician Professor Walter Willcox of Cornell. Willcox, who was at various times president of the American Statistical Association and the American Economic Association, argued that Webster's method is even-handed in its treatment of small and large states while Hill's is biased toward small states. Instead of relying on abstract mathematical arguments, as Huntington did, Willcox demonstrated this point by analyzing data. He prepared tables and diagrams showing how the small and the medium and the large states fared individually, and as groups, under each of the five methods. He concluded that "if the main purpose is, as it

probably was in the Constitutional Convention of 1787 to hold the balance even between the large and the small States as groups, that end is best secured by the method of [Webster]." U. S. Congress. House, *Apportionment of Representatives*, 1928, at 61, from a memorandum submitted by Walter F. Willcox, dated February 21, 1928; *Fair Representation* at 55.

49. Congress was understandably confused by these rival claims and turned for advice to the National Academy of Sciences. The Academy appointed a committee of four prominent mathematicians to study the matter. In their 1929 Report to Congress, the committee repeated Huntington's arguments: "There are five methods of apportionment now known which are unambiguous . . . and should be considered at this time. These five methods are [Adams, Dean, Hill, Webster, Jefferson]. In the present state of knowledge your committee regards these as the only methods of apportionment avoiding the so-called Alabama paradox which require consideration at this time. . . . Each method in the list favors the larger States as compared with the methods which precede it. This means in the case of the second [Dean] and fourth [Webster] methods, for example, that if for two unequal States A, B, the fourth method assigns more Representatives to A and fewer to B than the second method, then the State A is the larger of A and B. The method of [Dean] and the method of [Webster] are symmetrically situated on the list. . . . A similar symmetry exists for the methods of [Adams] and [Jefferson] for which the [measures of inequality] seem, however, more artificial than those for any one of the other three methods. The method of equal proportions is preferred by the committee because it satisfies the [relative difference] test . . . when applied either to sizes of congressional districts or to numbers of Representatives per person, and because it occupies mathematically a neutral position with respect to emphasis on larger and smaller States." This opinion was reiterated in a 1948 report by another National Academy of Sciences committee. Stipulations 69 & 70.

50. The claim of mathematical "neutrality" therefore amounted to nothing more than the observation that Hill's method ranked third in a list of five methods. This argument does not in fact demonstrate that Hill's method is unbiased. Moreover, it is not clear that Congress acted on or accepted this argument. In 1941, the adoption of Hill's method over Webster's had the effect of giving one more seat to Arkansas (in that period a predominantly Democratic state) and one less to Michigan, a state which in that period tended to vote Republican. Every Democrat in the House, except those from Michigan, voted for the change to Hill's method, and every Republican voted against.

51. An apportionment method is *biased* towards a state or class of states if, over many apportionments, it tends to give the residents of these states more than their fair share of representation on average. Similarly, a method is biased against a class of states if, over many apportionments, it tends to give their residents less than their fair share of representation on average.

52. Bias in an apportionment method only becomes apparent by looking at its results over many different censuses. A simple test of bias is the following. In each census, compute the percentage difference between the number of seats that each state received and the state's quota. This is the state's percentage deviation from the ideal. It is the same as the percentage difference between the state's average share of a representative and the national average share of a representative. For each state compute the average percentage deviation from the ideal over many different censuses. If the result is positive the state was favored on average, if negative it was disfavored on average. An apportionment method is unbiased if, over many censuses, the average percentage deviation from the ideal is close to zero for every state.

53. The twenty-one historical United States censuses from 1790 to 1990 provide a natural source of data to test for the presence of bias in various apportionment formulas. If Hill's method had been used throughout this period, the residents of Massachusetts would have received, on average, about .8 percent less representation than their ideal share. On the other hand, if Webster's method had been used throughout this period, the residents of Massachusetts would have received, on average, almost exactly their ideal share.

54. Massachusetts is not the only large state that, on average, would have been under-represented over the years under Hill's method. (Massachusetts received less than its quota of seats in five out of the six censuses since 1941. Stipulation 83.). Under Hill's method, the residents of the large states would have received less than the national average share and the residents of the small states would have received more than the national average share, over the twenty-one historical censuses on average. This is true even after allowing for the constitutional provision of a minimum of one seat per state.

55. To verify this proposition, I considered each census from 1790 to 1990 and divided the states into four categories. The "very small" states are those with a quota less than .5. The Constitution mandates that these states receive one seat each no matter how small their quotas, so they will necessarily be favored in any constitutional apportionment. After

setting these states aside, I divided the remaining states into three approximately equal-sized groups: large, medium, and small.[1]

56. In any given apportionment the large states are favored as a group if their residents' average share of a representative is more than the ideal (the national average share of a representative). They are disfavored as a group if their residents' average share of a representative is less than the ideal. Similarly the small states are favored as a group if their residents' average share of a representative is more than the ideal ; they are disfavored as a group if their residents' average share of a representative is less than the ideal.

57. Every method will sometimes favor the large states as a group and sometimes favor the small states as a group, depending on how the population counts happen to fall. An unbiased method will sometimes favor the small states and sometimes favor the large states, but over many apportionments these advantages and disadvantages will more or less balance out. A biased method, by contrast, will show a systematic tendency to favor one group or another over many apportionments.

58. Since it was adopted in 1941, Hill's method gave the residents of small states, on average, over 6 percent more representation per capita than it did to residents of large states. If Hill's method had been used in all twenty-one historical censuses from 1790 to 1990, it would have given, on average, over 3.3 percent more representation per capita to residents of small states than to residents of large states. These results provide concrete evidence that Hill's method is biased against the large states.

59. The state of Montana is proposing to replace Hill's method with either Dean's method or Adams' method. But these two methods are even more biased towards small states than Hill's. Over the twenty-one historical censuses, Dean's method would have given, on average, over 5 percent more representation per capita to the small states than to the large states. Adams' method would have given, on average, over 18 percent more representation per capita to the small states than to the large states.

60. The Webster method would have produced much more even-handed results than Adams, Dean, or Hill. If Webster's method had been used in all twenty-one historical

[1] If the number of states remaining after the very small states are deleted is not divisible by three, then the "medium" category of states takes up the extras.

censuses, then on average it would have given less than one-half of one percent more representation per capita to residents of small states as compared to residents of large states.

61. Moreover, *in every single census* the percentage difference between a person's share of a representative in large states and a person's share of a representative in small states would have been smaller, and at least not larger, under Webster's method than under any of the other three methods. In this sense, the Webster apportionments would have come at least as close, and often closer, to meeting the ideal of one person one vote than the Hill, Dean, or Adams apportionments.

62. These results are buttressed by theoretical calculations. It can be shown that, in a variety of statistical models, Webster's method is the least biased of all divisor methods. *Fair Representation* at 118-128. These same models show that Hill's method is biased toward the small states. The magnitude of this bias can be estimated theoretically. Under the current distribution of seats, Hill's method can be expected to give between 3 and 4 percent more representation per capita to the small states than to the large states over many apportionments. The Webster method can be expected to exhibit no discernible bias toward either the small states or the large states over many apportionments. Computer simulations support this observation. M. L. Balinski and H. P. Young, *Evaluation of Apportionment Methods*, report prepared under Contract No. CRS 84-15 for the Congressional Research Service of the Library of Congress, Washington, D.C, 1984. Such simulations would have been difficult if not impossible to carry out by the National Academy of Sciences committees in 1929 and 1948 because they lacked modern computing technology.

63. Thus theoretical calculations, computer simulations, and empirical evidence provided by historical census data all support the conclusion that Webster's method is substantially less biased than Hill's method.

VII. CONCLUSION

64. In summary, Webster's method is superior to Hill's method for three reasons. First, Webster apportionments bring each person's share of representation as near equality as possible no matter in what state they reside. Second, Webster apportionments bring the states as close to their shares (quotas) as possible: no state can be brought closer to its quota without moving some other state further from its quota, whether measured in absolute or percentage terms. Hill apportionments do not meet this standard. Indeed, it may be shown that Webster's method is the only divisor method that does meet this standard. Third, Webster's method comes closer to treating large states and small states evenhandedly. The empirical evidence based on twenty-one censuses is that Hill's method would have awarded, on average, over 3.3 percent more representation to residents of small states than to residents of large states. Over these twenty-one censuses Webster's method would have given, on average, less than half of one percent more representation to residents of small states than to residents of large states. Theoretical models suggest that, over a long series of apportionments, Hill's method will give on average between 3 and 4 percent more representation to residents of small states than to residents of large states, given the current distribution of seats among the fifty states. These same models suggest that, over a long series of apportionments, the Webster method would have no discernible bias in favor of either small or large states. *Fair Representation* at 127-8. Computer simulations over a thousand hypothetical apportionments support the proposition that Webster's method has no discernible bias toward small or large states, whereas Hill's method is biased toward small states. Balinski and Young report to the Congressional Research Service.

65. For these three reasons, Webster's method comes closer than any other method (and in particular closer than the methods of Hill, Dean, and Adams) to meeting the constitutional standard of one person one vote, and to achieving equal representation as nearly as practicable.

Signed under the penalties of perjury this 10th day of February, 1992.

H. Peyton Young

October, 1991

H. PEYTON YOUNG
Curriculum Vitae

PERSONAL

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B.A. - 1966, Cum Laude in General Studies, Harvard University
Ph.D. - 1970, Mathematics, University of Michigan

POSITIONS HELD

1982-Present	Professor of Public Policy and Applied Mathematics, University of Maryland, College Park, Maryland 20742
1976-1982	Research Scholar, International Institute for Applied Systems Analysis, Laxenburg, Austria. Deputy Chairman of the System and Decision Sciences Area
1971-1975	Graduate School of the City University of New York. Assistant, then Associate Professor of Mathematics
1971	National Water Commission, Arlington, Virginia

VISITING POSITIONS

1988-89	Guest Scholar, The Brookings Institution, Washington, DC
1987	Graduate School of Business, University of Chicago
1981	Center for Mathematics and Decision Theory, University of Paris
1978	University of Bonn

BOOKS

Fair Representation (with M.L. Balinski). New Haven, Conn.: Yale University Press, 1982.

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C. Natural Resources Economics

"Forecasting Water Use for Policy Making: A Review," (with R.G. Thompson), Water Resources Research 9 (1973), 729-99.

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D. Mathematics

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"The Evolution of Conventions," June 1991

"Sharing The Burden of Global Warming," August 1991

"An Evolutionary Model of Bargaining," September 1991

PREPARED TESTIMONY FOR GOVERNMENT AGENCIES

Subcommittee on Census and Population, United States House of Representatives -- Testimony on defects in the current formula for apportioning the House of Representatives, 1981.

Attorney General's Office of the State of Maryland -- Testimony on the gerrymandering of electoral districts, 1982.

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Subcommittee on Regulation and Information, Committee on Governmental Affairs, United States Senate -- Testimony on projection errors in the 1990 census and the apportionment of the United States House of Representatives, 1989.

RESEARCH GRANTS

Army Research Office, 1973-1974
National Science Foundation, 1975-1986, 1990-91
Office of Naval Research, 1986-1989
Russell Sage Foundation, 1989-1991

PROFESSIONAL ACTIVITIES

Associate Editor, Games and Economic Behavior, Social Choice and Welfare
Member, United States Advisory Committee for IIASA
Member, Econometric Society
Member, Operations Research Society of America
Member, American Economic Association
Member, American Political Science Association

UNITED STATES DISTRICT COURT
DISTRICT OF MASSACHUSETTS

COMMONWEALTH OF MASSACHUSETTS, EDWARD F. BERLIN and KAREN J. KEPLER, Plaintiffs, v. ROBERT MOSBACHER, as Secretary of the United States Department of Commerce; MICHAEL DARBY, as Undersecretary of Economic Affairs of the United States Department of Commerce; THE BUREAU OF THE CENSUS; BARBARA BRYANT, as Director of the Bureau of the Census; GEORGE HERBERT WALKER BUSH, as President of the United States; and DONNALD K. ANDERSON, as Clerk of the United States House of Representatives Defendants.	Civil Action No: 91-11234 WD

STIPULATION OF FACTS

The parties stipulate, for purposes of this litigation only, that the facts set forth in these stipulations are true. By so stipulating, the parties do not admit that anything set forth herein, or in any document referenced in or attached to this Stipulation, is relevant or material to any issue in this case. As to the documents attached as Exhibits to this Stipulation or referenced in the Administrative Record, the parties stipulate that those documents are true and authentic copies of the originals, were authored by the agencies and/or

THE STIPULATIONS RELATING TO FACTS WHICH ARE IRRELEVANT TO THE ISSUES BEFORE THIS COURT HAVE BEEN OMITTED.

to identify a home of record, legal residence, and last duty station. As with military personnel records, home of record was used first, then legal residence, and finally last duty station. Other federal agencies were instructed to determine the home states of their employees, and their dependents, stationed overseas, based solely on their administrative records.

58. In conducting the 1990 census, the Census Bureau did not require or verify that federal overseas employees and their dependents were apportioned to the state in which they had elected to vote. Nor did the Census Bureau require or verify that individuals in the United States were apportioned to the state in which they had elected to vote.

APPORTIONMENT FORMULAS

59. The Census Bureau conducts the decennial census to determine the whole number of persons in each state, and then calculates the apportionment of seats in the U.S. House of Representatives among the states. The calculation of the apportionment is based upon the method of equal proportions, also known as the Hill-Huntington method (hereinafter the "Hill method"), as required by Title 2, United States Code, Sections 2a(a), 2b. These results then are conveyed to the President through the Secretary of Commerce. The President then transmits to the United States Congress (the Congress) a statement of the whole number of persons in each state and the

number of representatives in the Congress to which each state is entitled, as ascertained under the decennial census. The Clerk of the U.S. House of Representatives (the House) then notifies each state (via a Certificate of Entitlement) of the number of representatives in Congress to which it is entitled.

60. The Congress has legislated four methods for apportionment since the first census of 1790. In 1941, Congress codified in Title 2 of the United States Code laws that govern the method currently used and the process for apportionment. Article I, Section 2 of the Constitution requires that Congress apportion House seats according to population, but does not allow for fractional House seats nor districts which cross state lines. Congress must apply some method to round off each state's otherwise fractional number of seats.

61. In 1792 Congress legislated use of the Jefferson method (also known as the method of greatest divisors) of apportionment after the first census in 1790. This method was used after each subsequent census through 1830. Under this method, the population of each state was divided by a fixed ratio (a pre-specified number of persons per representative) and the resulting integer was the size of the state's delegation, regardless of the size of the fractional remainder. For example, if the ratio for a particular state was 3.4, the state would receive 3 representatives. If the ratio was 3.9, the state still would receive 3

representatives. This method did not require that the size of the House, i.e., the total number of representatives, be a fixed number (in fact, the method would have to be modified for use with a fixed House size). Instead, the size of the House was the sum of the number of representatives given to each state using the above method. The size of the House grew over time based on apportionments following the 1790, 1800, 1810, 1820, and 1830 censuses.

62. Following the 1840 census, the Congress legislated use of the Webster method (also known as the method of major fractions (hereinafter "Webster method")) for the apportionment calculation. As then employed, with no mandate that the size of the House be fixed, it used a fixed ratio (of a pre-specified number of persons per representative), but an additional seat was assigned for each fractional remainder greater than one-half. For example, if the ratio for a state was 3.2, the state received 3 representatives. But if the ratio was 3.9 (or anything greater than 3.5, but less than 4.5), the state received 4 representatives. This method also did not require that the size of the House be a fixed number (and in fact would need to be modified for use with a fixed House size); the total number of representatives again was the sum of the representatives allocated to each state under this method.

63. Congress legislated use of the Vinton method (also known as the Hamilton method) for apportionment following the

censuses of 1850 through 1900. It utilized a fixed number of representatives. In this method, the apportionment population of the United States was divided by the fixed size of the House to derive a national-average-size Congressional district. The population of each state was then divided by this national-average-size Congressional district to obtain an exact quota of representatives. (This is known as the "House seat quota"). One representative was assigned to each state whose exact quota was less than one (to meet the Constitutional requirement that each state receive at least one representative). The other states (having quotas greater than or equal to 1.0) were assigned a seat for each whole number of their respective quotas, and any remaining seats were assigned on the basis of the highest fractional remainders for those states. Under this method, it is mathematically possible that a state could receive fewer seats if the size of the House were increased and the population of all states remained constant. This anomaly is known as the "Alabama paradox", because it was observed that for the 1880 census, Alabama would receive eight seats with a House size of 299 and seven seats with a House size of 300 by the use of the Vinton method.

64. In 1910, Congress legislated use of the Webster, or major fractions, method. Congress also fixed House membership at 433, with the proviso that "if the Territories of Arizona and New Mexico shall become States in the Union before the apportionment of Representatives under the next decennial

census they shall have one Representative each.... " 37 Stat. 14 (1911). Both Arizona and New Mexico became states in 1912 and House membership has been 435 since that time, except when it was increased temporarily when Alaska and Hawaii became states in 1959.

65. Congress could not decide on an apportionment plan following the 1920 census, but in 1929 passed a law that made reapportionment automatic, using whatever method was used for the previous apportionment in the event the Congress did not agree on another method.

66. Using results of the 1930 census, both the Hill method and the Webster method produced the same apportionment. The Congress did not legislate any method of apportionment, so as a result of the 1929 legislation, the apportionment for 1930 was automatic and was based upon the Webster method (because it had been used for the previous (1910) apportionment).

67. In January 1941, the President transmitted to Congress apportionments based upon both the Hill method and the Webster method. The methods produced the same apportionment except for two states, Arkansas and Michigan. The Hill method resulted in an additional seat for Arkansas; the Webster method resulted in an additional seat for Michigan. Congress debated the apportionment methodologies and enacted legislation adopting the Hill method, by a vote in which every Democrat (except those from Michigan) voted for, and every Republican voted against, the bill. No legislation affecting the method of

apportionment has been passed by Congress since 1941. As a result, the Hill method has been used for each subsequent apportionment, including 1990.

68. One of the products of the lengthy debates in the Congress regarding apportionment following the 1920 census, was the Congress' commissioning the National Academy of Sciences (NAS) to prepare a report "regarding the mathematical aspects of the problem of reapportionment". The report of the Committee on Apportionment discussed five methods "now known which are unambiguous (that is, lead to a workable solution)": smallest divisors (Adams method), harmonic means (Dean method), equal proportions (Hill method), major fractions (Webster method), and greatest divisors (Jefferson method). With a fixed House size, apportionments under these five methods are calculated using formulas involving each state's apportionment population and a divisor which determines each state's priority for its next seat (after one seat is assigned to each state). The divisors for the different formulas are functions of the number of seats already assigned to a given state which vary according to specific goals that each method is designed to achieve.

69. The report of the National Academy of Sciences committee, submitted to Congress in 1929, stated that the committee preferred the Hill method. The report analyzed five methods: Adams, Dean, Hill, Webster and Jefferson. It listed them in the order in which they "favor the larger States as

compared with the methods which precede it" (the Adams method favoring the small states the most, Jefferson's favoring the large states the most, and the Hill method listed third among the five); and stated that Hill's method "occupies mathematically a neutral position with respect to emphasis on larger and smaller states". (Report of the National Academy of Sciences Committee On Apportionment from the Annual Report of the National Academy of Sciences, Fiscal Year 1928-29, pp. 20-23).

70. In 1948, Congress asked the National Academy of Sciences to re-examine the various apportionment methods. An NAS committee concurred with the conclusion of the 1929 report.

71. Following both the 1970 and 1980 decennial censuses, Congress conducted hearings regarding methods of Congressional apportionment.

72. The apportionment method used in 1990, pursuant to Congressional legislation in force since 1941, was the Hill method. Using this method, Massachusetts received 10 seats under the apportionment based on the 1990 census.

73. If the Webster method for apportionment based on the 1990 census had been used (either excluding or including the counts of overseas military, other Federal employees, and dependents), Massachusetts would have received 11 seats. The Webster method may be applied as follows: a) A divisor would be found by dividing the apportionment population by 435 (the

total number of seats) to produce the national average size district. b) Each state's apportionment population would then be divided by the above divisor to obtain a quotient consisting of a whole number (possibly zero) and a fraction. c) If a state's quotient was less than 1.0, it would be rounded up to 1.0 no matter how small the fraction. d) For all other states (those with a quotient greater than or equal to 1.0), fractions less than one-half (0.5) would be rounded down, and fractions greater than one-half (0.5) would be rounded up. e) If the whole numbers resulting from this process summed to 435, the apportionment would be determined and no alteration of the divisor would be needed. f) If too many seats, or too few seats, were allocated by this process, the divisor would be made larger or smaller, respectively, until the total number of whole numbers achieved after rounding was equal to 435.

74. Under the Hill method, 1) each state first receives one Representative, as required under the Constitution. 2) A series of priority values is calculated for each state from which can be determined the state's entitlement to a second Representative, a third, and so on. 3) When the priority values are arranged in sequence (from highest to lowest), they indicate which state should get the 51st Representative, which the 52nd, etc. up through the 435th Representative. 4) The priority values are determined by multiplying the apportionment population of each state by the reciprocal of the geometric mean of the number of seats already assigned, a, and the next

integer, a+1, i.e., 1 divided by the square root of the quantity a(a+1). Alternatively, the apportionment under the Hill method could be obtained as described in paragraph 73 for the Webster method, with one-half (0.5) replaced by the following quantity: the square root a(a+1), minus a, where a is the largest integer not exceeding the state's quotient. For example, if the quotient is 2.7, then the square root of (2X3) minus 2 is approximately 0.449.

75. For quotients greater than one, the Hill rounding point is less than .50. The larger the whole number in a state's quotient, the larger is the Hill rounding point. For example, if a state's quotient falls between 1 and 2, then its Hill rounding point is the square root of 1 X 2, minus 1, which is approximately .414. If a state's quotient falls between 20 and 21, then its Hill rounding point is the square root of 20 X 21, minus 20, which is approximately .494.

76. For quotients greater than one, the Webster rounding point is always .50 exactly.

77. The rounding points in the previous two paragraphs are not always indicative of how the Webster and Hill methods would round the exact quota of a state. For the 1990 census, New York, New Jersey, Massachusetts, and Oklahoma have exact quotas of 31.521, 13.536, 10.532 and 5.516, respectively. The fractional portion of the quota for each of these four states is above the rounding points for both the Webster and the Hill methods. Yet, the Webster method rounds down the exact quota

for each of these states except Massachusetts and the Hill method rounds down the exact quota for each of these states except Oklahoma. Other examples exist in previous censuses where the exact quota is below the rounding point for both Webster and Hill and where either the Hill or Webster methods, or both, round up the exact quota.

78. If the Webster and Hill methods yield different apportionments, then the Hill method, as compared with the Webster method, awards fewer seats to the state or states with larger populations and more seats to the state or states with smaller populations. For example, in 1990, Webster's method gives one more seat to Massachusetts, which has a larger population, rather than to Oklahoma, which is given the extra seat under the Hill method.

79. Exhibit D displays the number of seats which would have been apportioned to Massachusetts based on the 1990 census under seven different apportionment methods. The number of seats that Massachusetts would receive varies depending on the method used and whether the overseas military, Federal employees, and dependents are included (Apportionment Population) or excluded (Resident Population).

80. By dividing the apportionment population by the total number of seats in the House of Representatives (435), a national-average-size congressional district can be obtained.

81. When the national-average-size congressional district is divided into the apportionment population of a state, the

mathematical result is that there is (except for the three smallest states) a whole number of congressional seats plus a fraction of a seat, known as the "House seat quota."

82. All methods of apportionment result in House seat quotas which contain fractions which must be rounded to whole numbers. Any rounding of a House seat quota results in some states receiving more seats than their House seat quota and some states receiving less seats than their House seat quota.

83. Exhibit E displays Massachusetts' House seat quota and the number of seats which would have been apportioned to it under the Hill and Webster methods from 1940 - 1990.

84. According to 1990 decennial census, Massachusetts was the thirteenth most populous state.

85. Massachusetts has been among the thirteen most populous states according to every decennial census since 1940.

86. In every one of the six apportionments from 1940-1990, the total number of seats allotted to the thirteen most populous states by the Hill method was less than the sum of their House seat quotas. In five out the six apportionments from 1940 - 1990, the total number of seats allotted to the thirteen most populous states by the Webster method was less than the sum of their House seat quotas.

87. Under the 1990 Census, Massachusetts will receive twelve electoral votes under the Hill method. Under the Webster method it would receive thirteen electoral votes.

DATED this 1st day of November, 1991.

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Exhibit D

POSSIBLE 1990 APPORTIONMENTS FOR MASSACHUSETTS UNDER VARIOUS METHODS

	EXACT Q	ADAM	HARM	EQPR	WEBS	JEFF	HAM
Apportionment Population	10.532	10	10	10	11	11	11
Resident Population	10.549	10	10	11	11	11	11

EXACT Q = EXACT QUOTA (The integer and fraction of seats due a state given its population)
ADAM = ADAMS (SMALLEST DIVISORS) METHOD - Never used
HARM = DEAN (HARMONIC MEAN) METHOD - Never used
EQPR = EQUAL PROPORTIONS (HILL) METHOD - Used 1940-1990
WEBS = WEBSTER (MAJOR FRACTIONS) METHOD - Used 1840, 1880-1910, 1930
JEFF = JEFFERSON (GREATEST DIVISORS) METHOD - Used 1790-1830
HAM = HAMILTON/VINTON METHOD - Used 1850-1870

Exhibit E

MASSACHUSETTS' CONGRESSIONAL ALLOCATION 1940-1990

	Quota	Hill	Webster
1940	14.333	14	14
1950	13.612	14	14
1960	12.543	12	13
1970	12.208	12	12
1980	11.049	11	11
1990	10.532	10	11

91-130 GOV

CRS Report for Congress

House Apportionment Following The 1990 Census: Using the Official Counts

David C. Huckabee
Analyst in American National Government
Government Division

February 4, 1991

Congressional Research Service • The Library of Congress

HOUSE APPORTIONMENT FOLLOWING THE 1990 CENSUS: USING THE OFFICIAL COUNTS

SUMMARY

An allocation of House seats based on official 1990 census State apportionment counts released by the Census Bureau on December 26, 1990, suggests that 19 seats will shift among 21 States if these totals are not altered by a decision to adjust the census for miscounts.

If no adjustment occurs or each State's share of the total apportionment count is not significantly altered from that experienced in the apportionment numbers, the gaining States will be: Arizona (+1), California (+7), Florida (+4), Georgia (+1), North Carolina (+1), Texas (+3), Virginia (+1), and Washington (+1). The losers will be: Illinois (-2), Iowa (-1), Kansas (-1), Kentucky (-1), Louisiana (-1), Massachusetts (-1), Michigan (-2), Montana (-1), New Jersey (-1), New York (-3), Ohio (-2), Pennsylvania (-2), and West Virginia (-1).

There is a slight possibility that the apportionment allocations might be altered if there is a statistical adjustment of the census to account for miscounts of people. The Secretary of Commerce has until July 15, 1991, to announce whether there will be such an adjustment.

Other ways the apportionment could change include the remote possibilities that the House size could be changed (permanently fixed by law at 435 since after the 1910 census), or the apportionment formula could be altered. Either or both these options would require changes in statutory law.

TABLE OF CONTENTS

HOUSE APPORTIONMENT FOLLOWING THE 1990 CENSUS: USING THE OFFICIAL COUNTS

INTRODUCTION

The release of the official census apportionment counts at the end of December 1990 provides the first opportunity to see how many House seats will be assigned to each State using official 1990 census figures, if there is no statistical adjustment for miscounts. [1] If no adjustment occurs, or each State's share of the total apportionment count is not significantly altered from that experienced in the apportionment numbers, the gaining States will be: Arizona (+1), California (+7), Florida (+4), Georgia (+1), North Carolina (+1), Texas (+3), Virginia (+1), and Washington (+1). The losers will be: Illinois (-2), Iowa (-1), Kansas (-1), Kentucky (-1), Louisiana (-1), Massachusetts (-1), Michigan (-2), Montana (-1), New Jersey (-1), New York (-3), Ohio (-2), Pennsylvania (-2), and West Virginia (-1).

In addition reporting which States were closest to gaining or losing additional Representatives using the official counts, this analysis also shows how the decision to include foreign-based Federal employees affected the apportionment as well. Data are also provided on other ways the apportionment could change including the remote possibilities that the House size could be altered, or the apportionment formula could be modified.

THE OFFICIAL APPORTIONMENT

Table 1 reports the House seat allocations based on the official apportionment numbers. The figures differ from the *resident population* of the States because the apportionment population includes Federal employees stationed overseas (including military personnel) and the resident population does not. These counts indicate that 19 seats will shift among 21 States—two more seats than the change following the 1980 census. A 19 seat shift is

[1] The "apportionment population" of the United States in 1990 includes all persons residing in the fifty States plus foreign-based military and civilian Federal employees. The 1990 Apportionment population of 249,022,783 includes 919,810 foreign-based Federal employees. The criteria by which these Federal workers were allocated to the States is described in: U.S. Library of Congress. Congressional Research Service. *The 1990 Census: Including Foreign-based Military and Civilian Personnel in the State Counts.* Report 90-305 GOV, by David C. Huckabee. Washington, 1990.

three to four more seats than was expected based on the most recent population projections prepared by the Census Bureau (early in 1990.) [2]

TABLE 1. House Apportionment Based on the Official 1990 Apportionment Population Counts

State	1980 apportionment population [a]	1990 apportionment population [b]	Percent difference from 1980	Seats (and change from 1980)
Alabama	3,890,061	4,062,608	4.25	7
Alaska	400,481	551,947	27.44	1
Arizona	2,717,866	3,677,985	26.10	6(+1)
Arkansas	2,285,513	2,362,239	3.25	4
California	23,668,562	29,839,250	20.68	52(+7)
Colorado	2,888,834	3,307,912	12.67	6
Connecticut	3,107,576	3,295,669	5.71	6
Delaware	595,225	668,696	10.99	1
Florida	9,739,992	13,003,362	25.10	23(+4)
Georgia	5,464,265	6,508,419	16.04	11(+1)
Hawaii	965,000	1,115,274	13.47	2
Idaho	943,935	1,011,986	6.72	2
Illinois	11,418,461	11,466,682	.42	20 (-2)
Indiana	5,490,179	5,564,228	1.33	10
Iowa	2,913,387	2,787,424	-4.52	5 (-1)
Kansas	2,363,208	2,485,600	4.92	4 (-1)
Kentucky	3,661,433	3,698,969	1.01	6 (-1)
Louisiana	4,203,972	4,238,216	.81	7 (-1)
Maine	1,124,660	1,233,223	8.80	2
Maryland	4,216,446	4,798,622	12.13	8
Massachusetts	5,737,037	6,029,051	4.84	10 (-1)
Michigan	9,258,344	9,328,784	.76	16 (-2)
Minnesota	4,077,148	4,387,029	7.06	8
Mississippi	2,520,638	2,586,443	2.54	5
Missouri	4,917,444	5,137,804	4.29	9

[2] The Census Bureau released four different population projections for 1990—each based on different assumptions. Series A and C indicated a 16 seat shift, whereas series B and D showed 15 seats. The data used to compute these apportionments was from: U.S. Dept. of Commerce. Bureau of the Census. *Projections of the Populations of States by Age, Sex and Race: 1989 to 2010*. Current Population Reports. Population Estimates and Projections. Series P-25, No 1053 by Signe I. Wetrogan. 1990.

TABLE 1. House Apportionment Based on the Official 1990 Apportionment Population Counts—Continued

State	1980 apportionment population [a]	1990 apportionment population [b]	Percent difference from 1980	Seats (and change from 1980)
Montana	786,690	803,655	2.11	1 (-1)
Nebraska	1,570,006	1,584,617	.92	3
Nevada	799,184	1,206,152	33.74	2
New Hampshire	920,610	1,113,915	17.35	2
New Jersey	7,364,158	7,748,634	4.96	13 (-1)
New Mexico	1,299,968	1,521,779	14.58	3
New York	17,557,288	18,044,505	2.70	31 (-3)
North Carolina	5,874,429	6,657,630	11.76	12(+1)
North Dakota	652,695	641,364	-1.77	1
Ohio	10,797,419	10,887,325	.83	19 (-2)
Oklahoma	3,025,266	3,157,604	4.19	6
Oregon	2,632,663	2,853,733	7.75	5
Pennsylvania	11,866,728	11,924,710	.49	21 (-2)
Rhode Island	947,154	1,005,984	5.85	2
South Carolina	3,119,208	3,505,707	11.02	6
South Dakota	690,178	699,999	1.40	1
Tennessee	4,590,750	4,896,641	6.25	9
Texas	14,228,383	17,059,805	16.60	30(+3)
Utah	1,461,037	1,727,784	15.44	3
Vermont	511,456	564,964	9.47	1
Virginia	5,346,279	6,216,568	14.00	11(+1)
Washington	4,130,163	4,887,941	15.50	9(+1)
West Virginia	1,949,644	1,801,625	-8.22	3 (-1)
Wisconsin	4,705,335	4,906,745	4.10	9
Wyoming	470,816	455,975	-3.25	1
Fifty State Total:	225,867,174	249,022,783	9.30	435 (19)

[a] The 1980 apportionment population is from: U.S. Dept. of Commerce. Bureau of the Census. 1980 Population and Number of Representatives by State. Memorandum. By Vincent P. Barabba. Dec. 31, 1981.

[b] The 1990 apportionment population is from: Barringer, Felicity. Census Bureau Places Population at 249.6 Million. New York Times, Dec. 27, 1990, p. A1. The percent differences and the apportionment totals were calculated by CRS. The 1990 apportionment population includes foreign-based Federal employees. The 1980 apportionment population does not.

THE APPORTIONMENT PROCESS

The reapportionment process for the House relies on rounding principles, but the actual procedure involves computing a "priority list" of seat assignments for the States. The constitution allocates the first fifty seats because each State must have at least one Representative. [3] A priority list assigns the remaining 385 seats (for a total of 435, see Appendix 1). The priority list method has been used by the Census Bureau to apportion the House since early in the 20th Century. It is a useful tool for identifying the States that nearly lost a House seat or those just missing gaining or retaining a seat. [4]

Table 2 extracts and enhances information from the priority list in Appendix 1 by showing which States were close to the 435th seat, and indicating approximately how many persons are needed by a State to either gain an additional seat (for States listed below sequence number 435) or lose a seat (those States above and including sequence number 435). For example, Table 2 shows that California's 52nd seat comes at priority sequence number 428 with 236,012 persons "to spare." These population additions or subtractions needed to gain or lose a seat assume that all other States' populations will remain constant--an unrealistic assumption given the likelihood that if one State's population is adjusted other States' populations will change as well. Thus the actual number of persons that California would need to receive 51 seats than 52, probably would not necessarily be 236,013 persons.

[3] Article I, Section 3, of the Constitution as modified by clause 2 of the Fourteenth Amendment, requires that "Representatives and direct taxes shall be apportioned among the several States . . . according to their respective numbers."

[4] The "priority list" rankings are calculated using the reciprocals of the geometric means of successive numbers ($1/\sqrt{n(n-1)}$), where "n" is the number of seats to be allocated to the State. The geometric mean (the square root of the product of two adjacent numbers) was chosen as the rounding point, rather than the arithmetic mean (the midpoint between two adjacent numbers), by the Congress in 1941. For a detailed explanation of how the apportionment formula works, see: U.S. Library of Congress. Congressional Research Service. *Apportioning Seats in the House of Representatives: The Method of Equal Proportions.* Report 88-143 GOV, by David C. Huckabee. Washington, 1988.

TABLE 2. Population Needed to Gain or Lose a Seat Using 1990 Census Apportionment Counts

Sequence	State	Official apport. population	Seat	Priority value	Pop. needed to gain or lose a seat	Percent of State pop.
420	NY	18,044,505	31	591,702.60	-514,022	2.85
421	CA	29,839,250	51	590,905.18	-810,890	2.72
422	OH	10,887,325	19	588,719.10	-256,537	2.36
423	IL	11,466,682	20	588,228.36	-260,847	2.27
424	IN	5,564,228	10	586,520.84	-110,746	1.99
425	MN	4,387,029	8	586,241.20	-85,265	1.94
426	PA	11,924,710	21	581,866.26	-143,849	1.21
427	NC	6,657,630	12	579,472.22	-53,138	.80
428	CA	29,839,250	52	579,430.15	-236,012	.79
429	TX	17,059,805	30	578,381.53	-104,249	.61
430	MS	2,586,443	5	578,346.15	-15,648	.61
431	WI	4,906,745	9	578,265.19	-29,003	.59
432	FL	13,003,362	23	578,069.92	-72,494	.56
433	TN	4,896,641	9	577,074.42	-18,899	.39
434	OK	3,157,604	6	576,496.87	-9,036	.29
435	WA	4,887,941	9	576,049.11	-10,199	.21
436	MA	6,029,051	11	574,847.17	12,606	.21
437	NJ	7,748,634	14	574,366.50	22,700	.29
438	NY	18,044,505	32	572,913.58	98,757	.55
439	KY	3,698,969	7	570,763.16	34,257	.93
440	CA	29,839,250	53	568,392.42	401,958	1.35
441	MT	803,655	2	568,269.89	11,001	1.37
442	AZ	3,619,064	7	567,525.26	55,241	1.50
443	GA	6,508,419	12	566,485.07	109,882	1.69
444	LA	4,238,216	8	566,355.23	72,542	1.71
445	MI	9,328,784	17	565,640.60	171,662	1.84
446	MD	4,798,622	9	565,522.77	89,319	1.86
447	IL	11,466,682	21	559,516.78	338,812	2.95
448	TX	17,059,805	31	559,413.02	507,333	2.97
449	OH	10,887,325	20	558,507.97	341,940	3.14
450	CA	29,839,250	54	557,767.31	978,034	3.28

Data calculated by CRS on the Library of Congress' mainframe computer.

OFFICIAL NOTIFICATION

Apportionment Numbers

Whatever the apportionment outcome, the law requires certain things to happen once the President is notified of the results by the Secretary of Commerce. On the first day (or within one week thereafter) of the Congress

following the decennial census the President is obligated to send the Congress a "statement showing the whole number of persons in each State" and an apportionment of House seats by the method of equal proportions. [5] Upon receipt of the President's message the Clerk of the House is directed to send "to the executive of each State a certificate of the number of Representatives to which [each] State is entitled" within fifteen calendar days after receiving the President's statement. [6]

Redistricting Numbers

Once the apportionment of House seats among the States is finished, States need to receive sufficiently detailed demographic data that they can draw new districts within the States. The Secretary of Commerce is directed to provide redistricting data "as expeditiously as possible after the decennial census" but no later than "one year after the decennial census date" (April 1). [7] The April 1, 1991, deadline gives most States at least a year to complete redistricting in time to meet the filing deadlines for House races in 1992, but twenty States have filing deadlines before April 1, 1992. If the Secretary of Commerce decides to adjust the census (which he may do up to July 15, 1991) the States will have less time to complete redistricting before the 1992 elections.

ACTIONS THAT COULD CHANGE THE APPORTIONMENT

Adjustment

Although the State populations reported in this analysis are the official apportionment populations submitted by the Department of Commerce to the President, these numbers may be revised. [8] The Census Bureau agreed in a July 1989 court stipulation to *consider* issuing figures adjusted for possible miscounts by July of 1991. If the figures are adjusted, there is a possibility that the apportionment could be affected besides shortening the period

[5] 2 *U.S.C.* 2(a). The President sent the apportionment results of the 1990 Census to the Congress on January 3, 1991 (the first day of the 102nd Congress).

[6] Ibid. The Clerk sent certificates showing how many Representatives each State was entitled to the Governors of each of the States on January 15, 1991.

[7] 13 *U.S.C.* 141(c).

[8] 13 *U.S.C.* 141(b) directs the Secretary of Commerce to report to the President within nine months of the Census date "the tabulation of total population by States . . . as required for the apportionment of Representatives in Congress among the several States."

available for the States for within-State redistricting. [9] The stipulation provides that census results will have a notice that reads: [10]

> *The population counts set forth herein are subject to possible correction for undercount or overcount. The United States Department of Commerce is considering whether to correct these counts and will publish corrected counts, if any, not later than July 15, 1991.*

The implications of a possible adjustment on the allocation of seats among the States is unclear. How, or if, such a revision would be made in the official apportionment totals has not yet been decided. The Secretary of Commerce has until July 15, 1991, to announce whether there will be such an adjustment.

Even if the population totals are adjusted, the apportionment may not change, but, the *order* in which the House seats are assigned to the States may be reshuffled. (Thus, if an adjustment occurs Washington *may not* be contending with Massachusetts for the last seat in the House as Table 2 and Appendix 1 suggest.) Small population differences can, and have, affected apportionments, however. In 1970, for example, fewer than 300 persons decided the last seat assigned in the House. (In theory, *one person* can make the difference.) In 1980, the last seat in the House was decided by 7,226 persons, and the seat shifted from Indiana to Florida after the release to the press of the unofficial State counts. Table 3 illustrates that the margin separating the States competing for the last seat in the House could have been nearly as close in 1990, as it was in 1970 if the Census Bureau had used the *resident* population rather than the resident population plus the foreign-based Federal employee component. Had the foreign-based federal civilian and military not been included in the official apportionment numbers, Massachusetts would have retained its 11th House seat, and Washington would not have gained its new 9th seat. Under this scenario, however, just

[9] This agreement to consider adjusting the census for miscounts was in response to *The City of New York, et al. v. United States Department of Commerce, et al.*, 88 Civ. 3474 (JMcL). The lawsuit was brought on November 3, 1988 in the U.S District Court for the Eastern District of New York. The suit was placed in abeyance when the plaintiffs and the Commerce Department agreed to a stipulation where the Commerce Department agreed to consider adjusting the Census.

[10] U.S. Library of Congress. Congressional Research Service. *Adjusting the 1990 Census for Miscounts: An Analysis of the Implications of the Stipulation Agreed to in the New York Court Case.* Testimony of Daniel Melnick and David Huckabee. In: U.S. Congress. House. Committee on Post Office and Civil Service. Subcommittee on Census and Population. Census Adjustment Lawsuit, 101st Cong., 1st Sess., October 17, 1989. U.S. Govt. Print. off., 1989., p. 99.

1,033 fewer persons in Massachusetts or 835 more persons in Washington would make the apportionment the same as the one using the official numbers.

TABLE 3. Population Needed to Gain or Lose a Seat Using 1990 Census Resident Counts

Sequence	State	State resident population	Seat	Priority value	Pop. needed to gain or lose a seat	Percent of State pop.
420	NY	17,990,455	31	589,930.23	-499,685	2.78
421	CA	29,760,021	51	589,336.21	-797,422	2.68
422	OH	10,847,115	19	586,544.80	-240,410	2.22
423	IL	11,430,602	20	586,377.49	-250,153	2.19
424	MN	4,375,099	8	584,646.98	-83,080	1.90
425	IN	5,544,159	10	584,405.39	-103,032	1.86
426	PA	11,881,643	21	579,764.81	-127,470	1.07
427	CA	29,760,021	52	577,891.65	-223,847	.75
428	NC	6,628,637	12	576,948.70	-39,107	.59
429	WI	4,891,769	9	576,500.25	-25,077	.51
430	TX	16,986,510	30	575,896.60	-69,365	.41
431	MS	2,573,216	5	575,388.50	-8,245	.32
432	FL	12,937,926	23	575,160.93	-36,352	.28
433	TN	4,877,185	9	574,781.51	-10,493	.22
434	OK	3,145,585	6	574,302.51	-4,150	.13
435	MA	6,016,425	11	573,643.33	-1,032	.02
436	WA	4,866,692	9	573,544.90	835	.02
437	NJ	7,730,188	14	572,999.20	8,690	.11
438	NY	17,990,455	32	571,197.50	77,034	.43
439	KY	3,685,296	7	568,653.37	32,339	.88
440	CA	29,760,021	53	566,883.22	354,890	1.19
441	AZ	3,665,228	7	565,556.82	52,407	1.43
442	MT	799,065	2	565,024.27	12,189	1.53
443	LA	4,219,973	8	563,917.41	72,782	1.72
444	GA	6,478,216	12	563,856.24	112,445	1.74
445	MI	9,295,297	17	563,610.15	165,471	1.78
446	MD	4,781,468	9	563,501.16	86,059	1.80
447	IL	11,430,602	21	557,756.26	325,588	2.85
448	TX	16,986,510	31	557,009.58	507,261	2.99
449	OH	10,847,115	20	556,445.24	335,253	3.09
450	CA	29,760,021	54	556,286.33	928,559	3.12

Data calculated by CRS on the Library of Congress' mainframe computer.

Changing the House Size

The U.S. Constitution (Art. 1, Sect. II) mandates that "Representatives shall be apportioned among the several States according to their respective numbers, counting the whole number of persons in each State." The requirement that districts must be apportioned *among* States means that district boundaries cannot cross State lines. The Constitution also sets a minimum size for the House of Representatives (one Representative for each State) and a maximum size for the House (one Representative for every 30,000 persons). Congress is free to choose a House size (with the concurrence of the President) within these boundaries. The House size increased in every decade except one in the 19th century to accommodate the growth in the country's population, but the permanent size of the House was fixed at 435 after the 1910 census.

If we set aside the issues of the efficacy of the proposal, changing the size of the House is the easiest means of significantly altering the allocation of seats among the States after a census. The 1990 census results provide a constitutional maximum of 8,301 seats (one for every 30,000 persons), and a minimum of 50 seats (one per State). If each State were to retain at least as many seats as it had after the 1980 census, the House size would have to be increased to at least 489. Table 3 displays the State-by-State results of a 489 seat apportionment. If such an increase were to occur: eleven States would gain an additional seat for the first time in comparison with a 435 seat apportionment (Alabama, Arkansas Colorado, Indiana, Maryland, Minnesota, Missouri, Oregon, South Carolina, Tennessee and Wisconsin); four States who would lose seats under a 435 seat apportionment would each gain a seat (Illinois, Massachusetts, New Jersey, and New York); nine States would retain seats they would have otherwise lost (Iowa, Kansas, Kentucky, Louisiana, Michigan, Montana, Ohio, Pennsylvania, and West Virginia); and the remaining 30 seats of the 54 seat increase would be divided among States already gaining seats in a 435 seat apportionment. A 489 seat House would lower the average size district of 572,466 to 509,249—9,986 fewer persons than the 1980 average of 519,234.

TABLE 4. House Apportionment Based on the Official 1990 Apportionment Population Counts Allocating 489 Seats (No States Lose Seats Compared to 1980)

		1990 allocation (change from 1980)	
	1980 seat total based on a 435 seat House	Seat total based on a 435 seat House	Seat total based on a 489 seat House
Alabama	7	7	8 (+1)
Alaska	1	1	1
Arizona	5	6 (+1)	7 (+2)
Arkansas	4	4	5 (+1)
California	45	52 (+7)	59 (+14)

TABLE 4. House Apportionment Based on the Official 1990 Apportionment Population Counts Allocating 489 Seats (No States Lose Seats Compared to 1980)—Continued

	1980 seat total based on a 435 seat House	1990 allocation (change from 1980) Seat total based on a 435 seat House	Seat total based on a 489 seat House
Colorado	6	6	7 (+1)
Connecticut	6	6	6
Delaware	1	1	1
Florida	19	23 (+4)	26 (+7)
Georgia	10	11 (+1)	13 (+3)
Hawaii	2	2	2
Idaho	2	2	2
Illinois	22	20 (-2)	23 (+1)
Indiana	10	10	11 (+1)
Iowa	6	5 (-1)	6
Kansas	5	4 (-1)	5
Kentucky	7	6 (-1)	7
Louisiana	8	7 (-1)	8
Maine	2	2	2
Maryland	8	8	9 (+1)
Massachusetts	11	10 (-1)	12 (+1)
Michigan	18	16 (-2)	18
Minnesota	8	8	9 (+1)
Mississippi	5	5	5
Missouri	9	9	10 (+1)
Montana	2	1 (-1)	2
Nebraska	3	3	3
Nevada	2	2	2
New Hampshire	2	2	2
New Jersey	14	13 (-1)	15 (+1)
New Mexico	3	3	3
New York	34	31 (-3)	35 (+1)
North Carolina	11	12 (+1)	13 (+2)
North Dakota	1	1	1
Ohio	21	19 (-2)	21
Oklahoma	6	6	6
Oregon	5	5	6 (+1)
Pennsylvania	23	21 (-2)	23
Rhode Island	2	2	2
South Carolina	6	6	7 (+1)

TABLE 4. House Apportionment Based on the Official 1990 Apportionment Population Counts Allocating 489 Seats (No States Lose Seats Compared to 1980)—Continued

	1980 seat total based on a 435 seat House	1990 allocation (change from 1980) Seat total based on a 435 seat House	Seat total based on a 489 seat House
South Dakota	1	1	1
Tennessee	9	9	10 (+1)
Texas	27	30 (+3)	34 (+7)
Utah	3	3	3
Vermont	1	1	1
Virginia	10	11 (+1)	12 (+2)
Washington	8	9 (+1)	10 (+2)
West Virginia	4	3 (-1)	4
Wisconsin	9	9	10 (+1)
Wyoming	1	1	1
Fifty State Total:	435	435 (19)	489 (54)

Data calculated by CRS on the Library of Congress' mainframe computer.

Table 5 illustrates how well the present apportionment formula and House size allocates seats. If Representatives were apportioned to the States in same proportion as the State's population relates to the population of the 50 States, each State's share of that population would match its share of the House. Given the law that limits the House size to 435, and the requirement that districts cannot cross State lines, some deviation is inevitable.

TABLE 5. Allocation of 435 House Seats Compared to Each State's Share of the Total Population of the Fifty States and the Amount that Each State's Average Size District Deviates From the National Average

State	Population: 1990 apportion-ment pop.	Population: Percent share of total	Seats: House seats (quota)[a]	Seats: Percent share of total	Average size district	Percent above or below national average
Alabama	4,062,608	1.63	7 (7.10)	1.61	580,373	1.36
Alaska	551,947	.22	1 (.96)	.23	551,947	-3.72
Arizona	3,677,985	1.48	6 (6.42)	1.38	612,998	6.61
Arkansas	2,362,239	.95	4 (4.13)	.92	590,560	3.06
California	29,839,250	11.98	52 (52.12)	11.95	573,832	.24

TABLE 5. Allocation of 435 House Seats Compared to Each State's Share of the Total Population of the Fifty States and the Amount that Each State's Average Size District Deviates From the National Average—Continued

	Population		Seats			
State	1990 apportionment pop.	Percent share of total	House seats (quota)[a]	Percent share of total	Average size district	Percent above or below national average
Colorado	3,307,912	1.33	6 (5.78)	1.38	551,319	-3.84
Connecticut	3,295,669	1.32	6 (5.76)	1.38	549,278	-4.22
Delaware	668,696	.27	1 (1.17)	.23	668,696	14.39
Florida	13,003,362	5.22	23 (22.71)	5.29	565,364	-1.26
Georgia	6,508,419	2.61	11 (11.37)	2.53	591,674	3.25
Hawaii	1,115,274	.45	2 (1.95)	.46	557,637	-2.66
Idaho	1,011,986	.41	2 (1.77)	.46	505,993	-13.14
Illinois	11,466,682	4.60	20 (20.03)	4.60	573,334	.15
Indiana	5,564,228	2.23	10 (9.72)	2.30	556,423	-2.88
Iowa	2,787,424	1.12	5 (4.87)	1.15	557,485	-2.69
Kansas	2,485,600	1.00	4 (4.34)	.92	621,400	7.87
Kentucky	3,698,969	1.49	6 (6.46)	1.38	616,495	7.14
Louisiana	4,238,216	1.70	7 (7.40)	1.61	605,459	5.45
Maine	1,233,223	.50	2 (2.15)	.46	616,612	7.16
Maryland	4,798,622	1.93	8 (8.38)	1.84	599,828	4.56
Massachusetts	6,029,051	2.42	10 (10.53)	2.30	602,905	5.05
Michigan	9,328,784	3.75	16 (16.30)	3.68	583,049	1.82
Minnesota	4,387,029	1.76	8 (7.66)	1.84	548,379	-4.39
Mississippi	2,586,443	1.04	5 (4.52)	1.15	517,289	-10.67
Montana	803,655	.32	1 (1.40)	.23	803,655	28.77
Nebraska	1,584,617	.64	3 (2.77)	.69	528,206	-8.38
Nevada	1,206,152	.48	2 (2.11)	.46	603,076	5.08
New Hampshire	1,113,915	.45	2 (1.95)	.46	556,958	-2.78
New Jersey	7,748,634	3.11	13 (13.54)	2.99	596,049	3.96
New Mexico	1,521,779	.61	3 (2.66)	.69	507,260	-12.85
New York	18,044,505	7.25	31 (31.52)	7.13	582,081	1.65
North Carolina	6,657,630	2.67	12 (11.63)	2.76	554,803	-3.18
North Dakota	641,364	.26	1 (1.12)	.23	641,364	10.74
Ohio	10,887,325	4.37	19 (19.02)	4.37	573,017	.10
Missouri	5,137,804	2.06	9 (8.97)	2.07	570,867	-.28

TABLE 5. Allocation of 435 House Seats Compared to Each State's Share of the Total Population of the Fifty States and the Amount that Each State's Average Size District Deviates From the National Average—Continued

	Population		Seats			
State	1990 apportionment pop.	Percent share of total	House seats (quota)[a]	Percent share of total	Average size district	Percent above or below national average
Oklahoma	3,157,604	1.27	6 (5.52)	1.38	526,267	-8.78
Oregon	2,853,733	1.15	5 (4.98)	1.15	570,747	-.30
Pennsylvania	11,924,710	4.79	21 (20.83)	4.83	567,843	-.81
Rhode Island	1,005,984	.40	2 (1.76)	.46	502,992	-13.81
South Carolina	3,505,707	1.41	6 (6.12)	1.38	584,285	2.02
South Dakota	699,999	.28	1 (1.22)	.23	699,999	18.22
Tennessee	4,896,641	1.97	9 (8.55)	2.07	544,071	-5.22
Texas	17,059,805	6.85	30 (29.80)	6.90	568,660	-.67
Utah	1,727,784	.69	3 (3.02)	.69	575,928	.60
Vermont	564,964	.23	1 (.99)	.23	564,964	-1.33
Virginia	6,216,568	2.50	11 (10.86)	2.53	565,143	-1.30
Washington	4,887,941	1.96	9 (8.54)	2.07	543,105	-5.41
West Virginia	1,801,625	.72	3 (3.15)	.69	600,542	4.68
Wisconsin	4,906,745	1.97	9 (2.97)	2.07	545,194	-5.00
Wyoming	455,975	.18	1 (.80)	.23	455,975	-25.55
Fifty States:	249,022,783		435		572,466	

[a]The House seat quota is calculated by dividing the national average size congressional district in 1990 of 572,466 into each State's apportionment population.

Table 5 illustrates that with a 435 seat House, when individual States' proportional shares of the total population are compared with their corresponding shares of the House, the differences appear to be trivial—ranging from -.11 (Washington, Oklahoma, and Mississippi) to +.12 (Massachusetts, New Jersey, and New York). If these differences are expressed proportionally, however, they assume greater importance—from -26 percent (Wyoming) to +29 percent (Montana). Overall, 347,690 persons separate the smallest congressional district (Wyoming at 455,965) from the largest (Montana at 803,655).

Even increasing the House size to 489 does little to change these figures. When the share of House figures are compared to the total population share numbers, the differences again appear trivial—ranging from -.11 (Iowa) to +.11

(Connecticut). The proportional differences are -27 percent (Montana) to +27 percent (South Dakota), with an 298,171 overall range separating smallest congressional district (Montana at 401,828) from the largest (South Dakota at 699,999).

Finding the best ways of minimizing these differences has long puzzled the Congress.

Changing the Apportionment Formula

The means by which the seats in the House are allocated among the States has been controversial from the beginning of the Republic. During the constitutional convention the decision to apportion seats among the States according to each State's population was part of the "Great Compromise" leading to approving the Constitution. An apportionment bill to enact a system devised by Alexander Hamilton was the first bill vetoed by George Washington.[11] Throughout the 19th century, and continuing into of the 20th, the manner of how allocations would be done was subject to congressional debate after each census. Finally in 1941, after 150 years of scrutiny, Congress accepted the "method of equal proportions" as the formula for apportioning House seats. This method (sometimes called the Hill or Huntington method) was adopted by 55 Stat 761 in November 1941. This law shifted a seat that had been allocated in January 1941 to Michigan, (from Michigan to Arkansas).

By enacting the equal proportions formula, the Congress did not end the controversy about the most appropriate method to allocate seats in the House. The main dispute is whether the current formula favors small States. A hearing was held on the topic before the House Subcommittee on the Census in 1981, but no further action was taken.[12] In 1982, two mathematicians, M.L. Balinski and H.P. Young, concluded that if "the intent is to eliminate any systematic advantage to either the small [State] or the large, then only one method, first proposed by Daniel Webster in 1832, will do."[13] This method, called the Webster method by Balinski and Young, is also referred to as the major fractions method.

The equal proportions formula had been endorsed in a 1929 report of the National Academy of Sciences prepared at the request of Speaker Longworth.

[11] M.L. Balinski and H.P. Young. *Fair Representation*. Yale University Press, New Haven and London. 1982, p. 21.

[12] U.S. Congress. House. Committee on Post Office and Civil Service. Subcommittee on Census and Population. *Census Activities and the Decennial Census*. Hearings, June 11, 1981. 97th Cong., 1st Sess. Washington, U.S. Govt. Print. Off., 1981.

[13] Ibid., p. 4.

The Academy concluded that "the method of equal proportions is preferred by the committee because it satisfies . . . [certain tests], and because it occupies mathematically a neutral position with respect to emphasis on larger and smaller States."[14] Balinksi's and Young's assessments of the major fractions method, and the National Academy's endorsement of equal proportions are obviously at odds. The differences partly reflect policy judgments, but in other respects the disagreement is explained by differing analytical techniques.[15]

Paradoxes and the Hamilton-Vinton Method

Why is there a controversy? Why not apportion the House the intuitive way by dividing each State's population by the national average size district (572,466 in 1990) and give each State its "quota" (rounding up at fractional remainders of .5 and above, and down for remainders less than .5). The problem with this proposal is that the House size would fluctuate around 435 seats. In some decades the House might include 435 seats, in others it might either be under or over the legal limit. In 1990, this method would result in a 438 seat House. One solution to this problem of too few or too many seats would be to divide each State's population by the national average size district, but instead of rounding at the .5 point, each State would receive the whole number of seats it would be entitled to after the division (except States entitled to less than one seat would receive one regardless). The fractional remainders would be ranked in order from largest to smallest. Seats would be assigned in rank order until 435 were allocated (see Table 6). If this system had been used in 1990, Massachusetts and New Jersey each would have retained seats that they lost, and Oklahoma and Mississippi each would have lost a seat.

This apportionment method was devised by Alexander Hamilton and later associated with Samuel Vinton. The Hamilton-Vinton method was mandated by law as the apportionment formula for the House from 1850 to 1900,[16] but it was never strictly followed because changes were made in the

[14] U.S. Congress. House. Committee on Post Office and Civil Service. Subcommittee on Census and Statistics. *The Decennial Population Census and Congressional Apportionment*. Report No. 91-1314, 91st Cong., 1st Sess., July 20, 1970. Washington, U.S. Govt. Print. Off., 1970. p. 21.

[15] For a fuller discussion of these issues see: U.S. Library of Congress Congressional Research Service. *Apportioning Seats in the House of Representatives: The Method of Equal Proportions*. Report 88-143 GOV, by David C. Huckabee. Washington, 1988.

[16] Laurence F. Schmeckebier. *Congressional Apportionment*. The Brookings Institution, Washington, 1941, p. 73.

apportionments that were not consistent with the method.[17] The Hamilton-Vinton method has simplicity in its favor, but its downfall was the *Alabama paradox*. Although the phenomenon had been observed previously, the "paradox" became an issue after the 1880 census when C.W. Seaton, Chief Clerk of the Census Office wrote the Congress on October 25, 1881, stating:

> While making these calculations I met with the so-called "Alabama" paradox where Alabama was allotted 8 Representatives out of a total of 299, receiving but 7 when the total became 300.[18]

Alabama lost its 8th seat when the House size was increased because of the vagaries of fractional remainders. With 299 seats Alabama's quota was 7.646 seats. It was allocated 8 seats based on this quota, but it was on the dividing point (see the Massachusetts example in table 6 below). When the House was increased in size to 300, Alabama's quota increased to 7.671, but Illinois and Texas now had larger fractional remainders than Alabama and they got extra seats, but the House had only increased in size by one seat. Thus, Alabama lost a seat.[19] This property of the Hamilton-Vinton method became a big enough issue that the formula was changed in 1911.

One could argue that the Alabama paradox should not be an important consideration in apportionments since the House size was fixed in size at 435, but the Hamilton-Vinton method is subject to other anomalies. Hamilton-Vinton is also subject to the *population paradox* and the *new States paradox*.

The population paradox occurs when a State that grows at a greater percentage rate than another has to give up a seat to the slower growing State. The new States paradox works in much the same way—at the next apportionment after a new State enters the Union the addition of the additional seats for the new State may result in seat shifts among States that otherwise would not have happened. Finding a formula that avoided the paradoxes was the goal when the Congress adopted a new method when the apportionment law was changed in 1911.

[17] Balinksi and Young., p. 37.

[18] Ibid., p. 38.

[19] Ibid., p. 39.

TABLE 6. Apportioning the House by Simple Rounding and Ranked Fractional Remainders (Hamilton-Vinton)

	"Quota" (State Pop. divided by 435)	Whole number of seats assigned	Rank of fractional remainders	Hamilton-Vinton allocation of seats	Simple rounding allocation of seats
Oregon	4.98	4.00	.98498	5.00	5.00
Missouri	8.97	8.00	.97486	9.00	9.00
Hawaii	1.95	1.00	.94819	2.00	2.00
New Hampshire	1.95	1.00	.94582	2.00	2.00
Iowa	4.87	4.00	.86915	5.00	5.00
Virginia	10.86	10.00	.85928	11.00	11.00
Pennsylvania	20.83	20.00	.83042	21.00	21.00
Texas	29.80	29.00	.80055	30.00	30.00
Colorado	5.78	5.00	.77835	6.00	6.00
Nebraska	2.77	2.00	.76805	3.00	3.00
Idaho	1.77	1.00	.76777	2.00	2.00
Rhode Island	1.76	1.00	.75728	2.00	2.00
Connecticut	5.76	5.00	.75697	6.00	6.00
Indiana	9.72	9.00	.71975	10.00	10.00
Florida	22.71	22.00	.71464	23.00	23.00
Minnesota	7.66	7.00	.66339	8.00	8.00
New Mexico	2.66	2.00	.65829	3.00	3.00
North Carolina	11.63	11.00	.62974	12.00	12.00
Wisconsin	8.57	8.00	.57124	9.00	9.00
Tennessee	8.55	8.00	.55359	9.00	9.00
Washington	8.54	8.00	.53839	9.00	9.00
New Jersey	13.54	13.00	.53553	14.00	14.00
Massachusetts	**10.53**	**10.00**	**.53172**	**11.00**	**11.00**
Last State rounded up a seat with a Hamilton-Vinton apportionment					
New York	31.52	31.00	.52065	31.00	32.00
Mississippi	4.52	4.00	.51807	4.00	5.00
Oklahoma	**5.52**	**5.00**	**.51579**	**5.00**	**6.00**
Last State rounded up a seat with by simple rounding					
Kentucky	6.46	6.00	.46146	6.00	6.00
Arizona	6.42	6.00	.42481	6.00	6.00
Montana	1.40	1.00	.40385	1.00	1.00
Louisiana	7.40	7.00	.40343	7.00	7.00
Maryland	8.38	8.00	.38237	8.00	8.00
Georgia	11.37	11.00	.36909	11.00	11.00
Kansas	4.34	4.00	.34192	4.00	4.00
Michigan	16.30	16.00	.29578	16.00	16.00
South Dakota	1.22	1.00	.22278	1.00	1.00
Delaware	1.17	1.00	.16810	1.00	1.00
Maine	2.15	2.00	.15423	2.00	2.00
West Virginia	3.15	3.00	.14713	3.00	3.00

TABLE 6. Apportioning the House by Simple Rounding and Ranked Fractional Remainders (Hamilton-Vinton)—Continued

	"Quota" (State Pop. divided by 435)	Whole number of seats assigned	Rank of fractional remainders	Hamilton-Vinton allocation of seats	Simple rounding allocation of seats
Arkansas	4.13	4.00	.12643	4.00	4.00
California	52.12	52.00	.12404	52.00	52.00
South Carolina	6.12	6.00	.12387	6.00	6.00
North Dakota	1.12	1.00	.12035	1.00	1.00
Nevada	2.11	2.00	.10694	2.00	2.00
Alabama	7.10	7.00	.09668	7.00	7.00
Illinois	20.03	20.00	.03032	20.00	20.00
Ohio	19.02	19.00	.01829	19.00	19.00
Utah	3.02	3.00	.01814	3.00	3.00
Vermont	.99	1.00	-.01310	1.00	1.00
Alaska	.96	1.00	-.03584	1.00	1.00
Wyoming	.80	1.00	-.20349	1.00	1.00
Total allocation:		412.00		435.00	438.00

Data calculated by CRS.

Avoiding the Paradoxes With Major Fractions

The major fractions method (used from 1911 until 1940) was a departure from methods previously considered because the divisor used to derive the apportionment was an artificial construct. Major fractions works much like the simple rounding method illustrated in table 6. It allocates seats by rounding up to the next seat when a State has a remainder of .5 and above. In order to work around the problem of a floating House size, major fractions uses a *sliding divisor*. First a trial divisor is found by dividing the population of the 50 States by 435. If 435 seats are allocated by using this trial figure (the national "ideal size" district rounded at the midpoint between each number) then no alteration of the divisor is necessary. But if too many seats are allocated (as would happen in 1990), the divisor is made larger (it *slides* up), if too few seats are apportioned, the divisor becomes smaller (it *slides* down). In 1990, if major fractions had been used rather than the current method, Massachusetts would have retained its 11th seat, and Oklahoma would have lost its 6th seat. The divisor used to apportion the seats in 1990 would have been any number from 574,110 to 574,195 (between 1,644 and 1,729 over the 1990 national average size district). One significant advantage of the major fractions as compared to simple rounding or the Hamilton-Vinton method is that it avoids the various paradoxes that plague the other two. Equal proportions (the current formula) also avoids the paradoxes.

TABLE 7. Population Needed to Gain or Lose a Seat Using Major Fractions Apportionment Method (1990 Census Apportionment Counts)

Sequence	State	State apportionment population	Seat	Priority value	Pop. needed to gain or lose a seat	Percent of State pop.
420	NY	18,044,505	31	591623.11	-534156	2.96
421	CA	29,830,250	51	590876.23	-846449	2.84
422	OH	10,887,325	19	588504.05	-266293	2.45
423	IL	11,466,682	20	588034.97	-271541	2.37
424	IN	5,564,228	10	585708.21	-110185	1.98
425	MN	4,387,029	8	584937.20	-81205	1.85
426	PA	11,924,710	21	581693.17	-155459	1.30
427	CA	29,830,250	52	579402.91	-272512	.91
428	NC	6,657,630	12	578924.34	-55367	.83
429	TX	17,059,805	30	578298.47	-123565	.72
430	FL	13,003,362	23	577927.20	-85891	.66
431	WI	4,906,745	9	577264.11	-26812	.55
432	TN	4,896,641	9	576075.41	-16708	.34
433	WA	4,887,941	9	575051.88	-8008	.16
434	MS	2,586,443	5	574765.11	-2949	.11
435	MA	6,029,051	11	574195.33	-898	.01
436	OK	3,157,604	6	574109.81	470	.01
437	NJ	7,748,634	14	573972.88	3003	.04
438	NY	18,044,505	32	572841.42	42648	.24
439	KY	3,698,969	7	569072.15	33301	.90
440	CA	29,830,250	53	568366.66	305913	1.03
441	GA	6,508,419	12	565949.47	94827	1.46
442	AZ	3,667,239	7	565843.84	54126	1.48
443	MI	9,328,784	17	565380.84	145439	1.56
444	LA	4,238,216	8	565095.46	68249	1.61
445	MD	4,798,622	9	564543.76	82038	1.71
446	IL	11,466,682	21	559350.34	304322	2.65
447	TX	17,059,805	31	559337.86	453153	2.66
448	OH	10,887,325	20	558324.35	309484	2.84
449	CA	29,830,250	54	557742.99	879935	2.95
450	NY	18,044,505	33	555215.53	616844	3.42

Data calculated by CRS on the Library of Congress' mainframe computer.

When major fractions is compared to the equal proportions method, one finds that major fractions minimizes the differences in representation in the House when those differences are expressed as each resident's share of a Representative on an *absolute* basis. Equal proportions minimizes the differences when they are compared on a *proportional* basis. Using Massachusetts and Oklahoma as the example in 1990, the first step in making

comparisons is to standardize the figures in some fashion. One way of doing this is to express each State's representation in the House as the number of Representatives per million residents that are assigned to each State.[20] When 11 seats are assigned to Massachusetts, and 5 are given to Oklahoma (using major fractions), Massachusetts has 1.824 Representatives per million persons and Oklahoma has 1.583 Representatives per million. The absolute difference between these numbers is .241 and the proportional difference between the two State's Representatives per million is 15.22 percent. When 10 seats are assigned to Massachusetts and 6 are assigned to Oklahoma (using equal proportions), Massachusetts has 1.659 Representatives per million and Oklahoma has 1.9 Representatives per million. The absolute difference between these numbers is .243 and the proportional difference is 14.53 percent.

Major fractions minimizes absolute differences, so in 1990 when assigning seats to Massachusetts and Oklahoma, they receive 11 and 5 seats respectively because the absolute difference (.241 Representatives per million) is smaller at 11 and 5 than it would be at 10 and 6 (.243). Equal proportions minimizes differences on a proportional basis, so it assigns 10 seats to Massachusetts and 6 to Oklahoma because the proportional difference between a 10 and 6 allocation (14.53 percent) is smaller than would occur with an 11 and 5 assignment (15.22 percent).

The Congress chose to minimize proportional differences when it selected the equal proportions method to allocate House seats in 1941.

Changing the Rounding Point With Equal Proportions

The only operational difference between a major fractions and the equal proportions apportionment (the method currently in use), is where the rounding occurs. Rather than rounding at the *arithmetic mean* that is always the midpoint (.5) between numbers, equal proportions rounds at the *geometric mean*. (The geometric mean is the square root of the multiplication of two numbers.) The equal proportions rounding point between 1 and 2, for example, is 1.414 (the square root of 2) rather than 1.5. The rounding point between 10 and 11 is the square root of 110 or 10.487. This property of the geometric mean (that larger numbers produce higher rounding points) is the intuitive basis for challenges to the equal proportions formula. The argument is that the equal proportions formula may not fairly allocate seats because larger States have to overcome ever increasing rounding thresholds to gain additional seats. This can be illustrated in the extreme by the examples of California and Montana in 1990. California received 52 seats based on its 1990 apportionment population, and Montana received just one seat. The rounding

[20] Representatives per million was computed by dividing the number of representatives assigned to the State by the State's population (which gives the number of representatives per person) and then multiplying the resulting dividend by 1,000,000.

point that California would need to equal or exceed to gain a 53rd seat in the House is 53.498 as contrasted to Montana's threshold of 1.414 to retain its 2nd seat.

The House has only been reapportioned twenty times since 1790. The equal proportions method has been used in five apportionments, and major fractions in two. Seven apportionments do not provide enough historical information to enable policy makers to generalize about the impact of differing methods. Computers, however can simulate reality by using random numbers to test many different hypothetical situations. These techniques (such as the "Monte Carlo" simulation method) are a useful way of observing the behavior of systems when experience does provide enough information to generalize about them.

Apportioning the House can be viewed as a system with four main variables: (1) the size of the House: (2) the population of the States; (3) the number of States; and (4) the method of apportionment. A 1984 exercise prepared for CRS involving 1,000 simulated apportionments examined the results when two of these variables were changed—the method and the State populations. Major fractions, equal proportions, and the Hamilton-Vinton methods were tested (along with several other alternatives). There was no discernible pattern by size of State in the results of the Hamilton-Vinton or major fractions apportionments.[21] The equal proportions formula was found to have "a definite tendency to give large States less than their fair shares [of seats] and the small States more."[22]

Adhering to quota, however, is not the only goal that the Congress endorsed when it adopted the equal proportions formula in 1941. The concept of minimizing *proportional* differences of district sizes of among States was deemed to be important as well, and no method does this as well as equal proportions.

CONCLUSION

If history proves to be prologue, in the absence of an adjustment for miscounts of persons the likelihood of change in the official apportionment allocations set out in this report is slim.

[21] H.P. Young and M.L. Balinski. *Evaluation of Apportion Methods*. Prepared under a contract for the Congressional Research Service of the Library of Congress. (Contract No. CRS84-15) Washington, 1984.

A modified version of Hamilton-Vinton was evaluated (which set all States with quotas less than one seat to one) because of the constitutional requirement that each State must receive at least one House seat.

[22] Ibid., p. 8.

There has been no increase in the House size since immediately after the 1910 census (with the brief temporary exception when Alaska and Hawaii were admitted). Advocates of changing the apportionment formula will face the daunting task of persuading the Congress and the President of the need to alter a method that has been relatively noncontroversial since it was adopted. Further obstacles include the difficulty of debating this technical matter, and adequately explaining the problems and benefits associated with the alternative methods.

One should not assume, however, that the matter of the best House apportionment method has been settled. Each alternative has its strengths and weaknesses. Whether the goal is simplicity (simple rounding or Hamilton-Vinton), or minimizing absolute differences (major fractions) or proportional differences (equal proportions), one can argue that the choice of a method is a policy decision rather than one to be left strictly to mathematicians.

Appendix 1 shows assignments of House seats based on the census apportionment counts. The appendix provides the full listing that formed the basis for Table 2.

APPENDIX 1. Apportionment Priority List Based on 1990 Census Apportionment Counts

Sequence	State	Seat	Priority value
The first 50 seats are assigned one each to every State by the Constitutional requirement that each State must have at least one seat.			
51	California	2	21,099,535.65
52	New York	2	12,759,391.63
53	California	3	12,181,821.46
54	Texas	2	12,063,103.59
55	Florida	2	9,194,765.29
56	California	4	8,613,849.35
57	Pennsylvania	2	8,432,043.16
58	Illinois	2	8,108,168.46
59	Ohio	2	7,698,501.20
60	New York	3	7,366,637.51
61	Texas	3	6,964,635.46
62	California	5	6,672,258.17
63	Michigan	2	6,596,446.31
64	New Jersey	2	5,479,111.55
65	California	6	5,447,875.79
66	Florida	3	5,308,599.72
67	New York	4	5,208,999.81
68	Texas	4	4,924,741.41
69	Pennsylvania	3	4,868,241.93
70	North Carolina	2	4,707,655.23
71	Illinois	3	4,681,252.81
72	California	7	4,604,295.11
73	Georgia	2	4,602,147.13
74	Ohio	3	4,444,731.33
75	Virginia	2	4,395,777.31
76	Massachusetts	2	4,263,182.77
77	New York	5	4,034,873.39
78	California	8	3,987,436.09
79	Indiana	2	3,934,503.28
80	Texas	5	3,814,687.81
81	Michigan	3	3,808,459.70
82	Florida	4	3,753,747.20
83	Missouri	2	3,632,975.98
84	California	9	3,516,587.79
85	Wisconsin	2	3,469,592.60
86	Tennessee	2	3,462,447.99
87	Washington	2	3,456,296.16
88	Pennsylvania	4	3,442,367.20

See notes at the end of the table.

APPENDIX 1. Apportionment Priority List Based on 1990 Census Apportionment Counts—Continued

Sequence	State	Seat	Priority value
89	Maryland	2	3,393,138.09
90	Illinois	4	3,310,145.91
91	New York	6	3,294,460.21
92	New Jersey	3	3,163,366.23
93	California	10	3,145,331.61
94	Ohio	4	3,142,899.95
95	Texas	6	3,114,679.44
96	Minnesota	2	3,102,097.90
97	Louisiana	2	2,996,871.22
98	Florida	5	2,907,639.71
99	Alabama	2	2,872,697.61
100	California	11	2,845,059.46
101	New York	7	2,784,326.89
102	North Carolina	3	2,717,965.76
103	Michigan	4	2,692,987.92
104	Pennsylvania	5	2,666,445.82
105	Georgia	3	2,657,050.63
106	Texas	7	2,632,384.41
107	Kentucky	2	2,615,566.01
108	Arizona	2	2,600,728.09
109	California	12	2,597,172.96
110	Illinois	5	2,564,027.67
111	Virginia	3	2,537,902.98
112	South Carolina	2	2,478,909.15
113	Massachusetts	3	2,461,349.49
114	Ohio	5	2,434,479.52
115	New York	8	2,411,297.55
116	California	13	2,389,051.45
117	Florida	6	2,374,077.80
118	Colorado	2	2,339,046.96
119	Connecticut	2	2,330,389.85
120	Texas	8	2,279,711.53
121	Indiana	3	2,271,586.31
122	New Jersey	4	2,236,837.92
123	Oklahoma	2	2,232,763.16
124	California	14	2,211,830.60
125	Pennsylvania	6	2,177,143.82
126	New York	9	2,126,564.37
127	Missouri	3	2,097,499.46
128	Illinois	6	2,093,519.75
129	Michigan	5	2,085,979.21
130	California	15	2,059,102.28
131	Oregon	2	2,017,893.92

See notes at the end of the table.

APPENDIX 1. Apportionment Priority List Based on 1990 Census Apportionment Counts—Continued

Sequence	State	Seat	Priority value
132	Texas	9	2,010,516.41
133	Florida	7	2,006,461.82
134	Wisconsin	3	2,003,170.03
135	Tennessee	3	1,999,045.09
136	Washington	3	1,995,493.33
137	Ohio	6	1,987,744.13
138	Iowa	2	1,971,006.37
139	Maryland	3	1,959,029.01
140	California	16	1,926,114.17
141	North Carolina	4	1,921,892.20
142	New York	10	1,902,056.92
143	Georgia	4	1,878,818.69
144	Pennsylvania	7	1,840,022.25
145	Mississippi	2	1,828,891.35
146	California	17	1,809,270.25
147	Texas	10	1,798,260.48
148	Virginia	4	1,794,568.57
149	Minnesota	3	1,790,996.89
150	Illinois	7	1,769,347.01
151	Kansas	2	1,757,584.58
152	Massachusetts	4	1,740,437.07
153	Florida	8	1,737,646.72
154	New Jersey	5	1,732,646.98
155	Louisiana	3	1,730,244.24
156	New York	11	1,720,475.20
157	California	18	1,705,796.31
158	Michigan	6	1,703,194.83
159	Ohio	7	1,679,950.30
160	Arkansas	2	1,670,355.18
161	Alabama	3	1,658,552.58
162	Texas	11	1,626,587.79
163	California	19	1,613,521.84
164	Indiana	4	1,606,254.23
165	Pennsylvania	8	1,593,505.83
166	New York	12	1,570,572.33
167	Florida	9	1,532,460.23
168	Illinois	8	1,532,299.29
169	California	20	1,530,721.18
170	Kentucky	3	1,510,097.60
171	Arizona	3	1,501,530.92
172	North Carolina	5	1,488,691.10
173	Texas	12	1,484,865.21
174	Missouri	4	1,483,156.23

See notes at the end of the table.

APPENDIX 1. Apportionment Priority List Based on 1990 Census Apportionment Counts—Continued

Sequence	State	Seat	Priority value
175	California	21	1,456,006.30
176	Georgia	5	1,455,326.51
177	Ohio	8	1,454,879.48
178	New York	13	1,444,716.30
179	Michigan	7	1,439,462.27
180	South Carolina	3	1,431,198.73
181	Wisconsin	4	1,416,455.24
182	New Jersey	6	1,414,700.28
183	Tennessee	4	1,413,538.47
184	Washington	4	1,411,027.00
185	Pennsylvania	9	1,405,339.93
186	Virginia	5	1,390,066.66
187	California	22	1,388,247.47
188	Maryland	4	1,385,242.82
189	Florida	10	1,370,674.05
190	Texas	13	1,365,877.22
191	Illinois	9	1,351,360.84
192	Colorado	3	1,350,449.27
193	Massachusetts	5	1,348,136.59
194	Connecticut	3	1,345,451.08
195	New York	14	1,337,546.63
196	California	23	1,326,516.39
197	Oklahoma	3	1,289,086.29
198	Ohio	9	1,283,082.99
199	West Virginia	2	1,273,941.23
200	California	24	1,270,042.73
201	Minnesota	4	1,266,426.16
202	Texas	14	1,264,555.87
203	Pennsylvania	10	1,256,974.20
204	Michigan	8	1,246,610.75
205	New York	15	1,245,188.18
206	Indiana	5	1,244,199.02
207	Florida	11	1,239,821.31
208	Louisiana	4	1,223,467.55
209	Utah	2	1,221,727.76
210	California	25	1,218,182.21
211	North Carolina	6	1,215,511.15
212	Illinois	10	1,208,693.83
213	New Jersey	7	1,195,639.89
214	Georgia	6	1,188,269.08
215	Texas	15	1,177,237.47
216	Alabama	4	1,172,773.89
217	California	26	1,170,391.58

See notes at the end of the table.

APPENDIX 1. Apportionment Priority List Based on 1990 Census Apportionment Counts—Continued

Sequence	State	Seat	Priority value
218	Oregon	3	1,165,031.49
219	New York	16	1,164,767.10
220	Missouri	5	1,148,847.73
221	Ohio	10	1,147,624.27
222	Iowa	3	1,137,960.95
223	Pennsylvania	11	1,136,975.93
224	Virginia	6	1,134,984.63
225	Florida	12	1,131,797.21
226	California	27	1,126,209.87
227	Nebraska	2	1,120,493.40
228	Texas	16	1,101,205.03
229	Massachusetts	6	1,100,748.87
230	Michigan	9	1,099,407.25
231	Wisconsin	5	1,097,181.37
232	Tennessee	5	1,094,922.05
233	New York	17	1,094,108.80
234	Illinois	11	1,093,304.69
235	Washington	5	1,092,976.67
236	California	28	1,085,243.01
237	New Mexico	2	1,076,060.23
238	Maryland	5	1,073,004.34
239	Kentucky	4	1,067,800.35
240	Arizona	4	1,061,742.79
241	Mississippi	3	1,055,910.81
242	California	29	1,047,152.30
243	Florida	13	1,041,101.93
244	Ohio	11	1,038,065.20
245	Pennsylvania	12	1,037,912.62
246	New Jersey	8	1,035,454.40
247	Texas	17	1,034,402.59
248	New York	18	1,031,535.64
249	North Carolina	7	1,027,294.36
250	Indiana	6	1,015,884.21
251	Kansas	3	1,014,741.83
252	South Carolina	4	1,012,010.42
253	California	30	1,011,645.28
254	Georgia	7	1,004,270.60
255	Illinois	12	998,046.41
256	Michigan	10	983,339.70
257	Minnesota	5	980,969.36
258	California	31	978,467.51
259	New York	19	975,735.07
260	Texas	18	975,244.09

See notes at the end of the table.

APPENDIX 1. Apportionment Priority List Based on 1990 Census Apportionment Counts—Continued

Sequence	State	Seat	Priority value
261	Arkansas	3	964,379.92
262	Florida	14	963,872.55
263	Virginia	7	959,237.03
264	Colorado	4	954,911.92
265	Pennsylvania	13	954,740.67
266	Connecticut	4	951,377.67
267	Louisiana	5	947,693.77
268	Ohio	12	947,619.86
269	California	32	947,397.10
270	Missouri	6	938,030.21
271	Massachusetts	7	930,302.53
272	New York	20	925,663.55
273	Texas	19	922,488.60
274	California	33	918,239.42
275	Illinois	13	918,069.09
276	New Jersey	9	913,184.87
277	Oklahoma	4	911,521.74
278	Alabama	5	908,426.63
279	Florida	15	897,316.53
280	Wisconsin	6	895,844.81
281	Tennessee	6	894,000.08
282	Washington	6	892,411.68
283	California	34	890,823.07
284	North Carolina	8	889,662.91
285	Michigan	11	889,464.22
286	Pennsylvania	14	883,917.61
287	New York	21	880,481.68
288	Maryland	6	876,104.34
289	Texas	20	875,149.50
290	Maine	2	872,020.33
291	Ohio	13	871,683.42
292	Georgia	8	869,723.76
293	California	35	864,996.63
294	Indiana	7	858,578.81
295	Nevada	2	852,878.24
296	Illinois	14	849,966.34
297	California	36	840,625.60
298	New York	22	839,506.30
299	Florida	16	839,362.91
300	Texas	21	832,433.24
301	Virginia	8	830,723.54
302	Kentucky	5	827,114.49
303	Oregon	4	823,801.74

See notes at the end of the table.

APPENDIX 1. Apportionment Priority List Based on 1990 Census Apportionment Counts—Continued

Sequence	State	Seat	Priority value
304	Pennsylvania	15	822,882.53
305	Arizona	5	822,422.32
306	California	37	817,590.39
307	New Jersey	10	816,777.34
308	Michigan	12	811,966.30
309	Ohio	14	807,021.57
310	Massachusetts	8	805,665.54
311	Iowa	4	804,659.98
312	New York	23	802,176.05
313	Minnesota	6	800,958.10
314	California	38	795,784.05
315	Texas	22	793,693.91
316	Missouri	7	792,780.17
317	Illinois	15	791,275.62
318	Hawaii	2	788,617.79
319	Florida	17	788,444.61
320	New Hampshire	2	787,656.83
321	North Carolina	9	784,608.87
322	South Carolina	5	783,899.80
323	California	39	775,110.76
324	Louisiana	6	773,788.69
325	Pennsylvania	16	769,736.26
326	New York	24	768,025.08
327	Georgia	9	767,024.19
328	Texas	23	758,400.80
329	Wisconsin	7	757,127.00
330	Tennessee	7	755,567.92
331	California	40	755,484.48
332	Washington	7	754,225.48
333	Ohio	15	751,296.22
334	Michigan	13	746,900.30
335	Mississippi	4	746,641.76
336	Indiana	8	743,550.98
337	Florida	18	743,352.69
338	Alabama	6	741,727.21
339	Maryland	7	740,443.26
340	Illinois	16	740,170.70
341	Colorado	5	739,671.50
342	New Jersey	11	738,802.90
343	Connecticut	5	736,933.88
344	California	41	736,827.74
345	New York	25	736,663.79
346	West Virginia	3	735,510.24

See notes at the end of the table.

APPENDIX 1. Apportionment Priority List Based on 1990 Census Apportionment Counts—Continued

Sequence	State	Seat	Priority value
347	Virginia	9	732,629.24
348	Texas	24	726,113.47
349	Pennsylvania	17	723,041.73
350	California	42	719,070.17
351	Kansas	4	717,530.90
352	Idaho	2	715,582.15
353	Rhode Island	2	711,338.09
354	Massachusetts	9	710,530.16
355	New York	26	707,763.66
356	Oklahoma	5	706,061.61
357	Utah	3	705,364.78
358	Florida	19	703,141.28
359	Ohio	16	702,773.39
360	California	43	702,148.53
361	North Carolina	10	701,775.48
362	Texas	25	696,463.58
363	Illinois	17	695,269.70
364	Michigan	14	691,494.92
365	Missouri	8	686,567.69
366	Georgia	10	686,047.27
367	California	44	686,005.00
368	Arkansas	4	681,919.64
369	Pennsylvania	18	681,690.26
370	New York	27	681,045.92
371	Minnesota	7	676,933.10
372	Kentucky	6	675,336.13
373	New Jersey	12	674,431.92
374	Arizona	6	671,504.99
375	California	45	670,587.24
376	Texas	26	669,140.55
377	Florida	20	667,058.37
378	Ohio	17	660,141.03
379	New York	28	656,272.29
380	California	46	655,847.22
381	Indiana	9	655,750.27
382	Wisconsin	8	655,691.14
383	Illinois	18	655,506.55
384	Virginia	10	655,283.49
385	Tennessee	8	654,340.94
386	Louisiana	7	653,970.76
387	Washington	8	653,178.36
388	Nebraska	3	646,917.11
389	Pennsylvania	19	644,814.46

See notes at the end of the table.

APPENDIX 1. Apportionment Priority List Based on 1990 Census Apportionment Counts—Continued

Sequence	State	Seat	Priority value
390	Texas	27	643,880.82
391	Michigan	15	643,746.75
392	California	47	641,741.37
393	Maryland	8	641,242.61
394	South Carolina	6	640,051.48
395	Oregon	5	638,114.00
396	Massachusetts	10	635,517.47
397	North Carolina	11	634,779.80
398	Florida	21	634,499.09
399	New York	29	633,237.93
400	California	48	628,229.44
401	Alabama	7	626,873.87
402	Iowa	5	623,286.86
403	Ohio	18	622,386.91
404	New Mexico	3	621,263.60
405	Georgia	11	620,553.10
406	Texas	28	620,459.09
407	New Jersey	13	620,387.08
408	Illinois	19	620,047.14
409	California	49	615,274.87
410	New York	30	611,765.99
411	Pennsylvania	20	611,724.70
412	Missouri	9	605,495.74
413	Florida	22	604,971.11
414	Colorado	6	603,939.23
415	California	50	602,843.86
416	Michigan	16	602,170.06
417	Connecticut	6	601,703.97
418	Texas	29	598,681.74
419	Virginia	11	592,726.21
420	New York	31	591,702.60
421	California	51	590,905.18
422	Ohio	19	588,719.10
423	Illinois	20	588,228.36
424	Indiana	10	586,520.84
425	Minnesota	8	586,241.20
426	Pennsylvania	21	581,866.26
427	North Carolina	12	579,472.22
428	California	52	579,430.15
429	Texas	30	578,381.53
430	Mississippi	5	578,346.15
431	Wisconsin	9	578,265.19
432	Florida	23	578,069.92

See notes at the end of the table.

APPENDIX 1. Apportionment Priority List Based on 1990 Census Apportionment Counts—Continued

Sequence	State	Seat	Priority value
433	Tennessee	9	577,074.42
434	Oklahoma	6	576,496.87
435	Washington	9	576,049.11
	The 435th seat is the last one provided by law		
436	Massachusetts	11	574,847.17
437	New Jersey	14	574,366.50
438	New York	32	572,913.58
439	Kentucky	7	570,763.16
440	California	53	568,392.42
441	Montana	2	568,269.89
442	Arizona	7	567,525.26
443	Georgia	12	566,485.07
444	Louisiana	8	566,355.23
445	Michigan	17	565,640.60
446	Maryland	9	565,522.77
447	Illinois	21	559,516.78
448	Texas	31	559,413.02
449	Ohio	20	558,507.97
450	California	54	557,767.31
451	Kansas	5	555,796.97
452	New York	33	555,281.24
453	Pennsylvania	22	554,787.68
454	Florida	24	553,459.80
455	California	55	547,532.16
456	Alabama	8	542,888.63
457	Texas	32	541,649.33
458	Missouri	10	541,571.83
459	Virginia	12	541,082.71
460	South Carolina	7	540,942.20
461	New York	34	538,701.92
462	California	56	537,665.94
463	New Jersey	15	534,706.13
464	Illinois	22	533,478.29
465	Michigan	18	533,291.06
466	North Carolina	13	533,036.87
467	Ohio	21	531,247.06
468	Florida	25	530,860.00
469	Indiana	11	530,528.06
470	Pennsylvania	23	530,117.99
471	Arkansas	5	528,212.62
472	California	57	528,148.99
473	Texas	33	524,979.20
474	Massachusetts	12	524,761.45

See notes at the end of the table.

APPENDIX 1. Apportionment Priority List Based on 1990 Census Apportionment Counts—Continued

Sequence	State	Seat	Priority value
475	New York	35	523,084.05
476	Georgia	13	521,090.43
477	Oregon	6	521,017.88
478	West Virginia	4	520,084.33
479	California	58	518,963.07
480	Wisconsin	10	517,216.08
481	Minnesota	9	517,016.09
482	Tennessee	10	516,151.03
483	Washington	10	515,233.97
484	Colorado	7	510,421.77
485	California	59	510,091.18
486	Florida	26	510,033.77
487	Illinois	23	509,756.16
488	Texas	34	509,304.61
489	Iowa	6	508,911.57
490	Connecticut	7	508,532.64
491	New York	36	508,346.32
492	Pennsylvania	24	507,549.32
493	Ohio	22	506,524.17
494	Maryland	10	505,818.92
495	Michigan	19	504,442.86
496	Maine	3	503,461.12
497	California	60	501,517.64
498	New Jersey	16	500,171.88
499	Louisiana	9	499,478.32
500	Utah	4	498,768.26

The 1990 census apportionment numbers that formed the basis for the calculation of this priority table are from: Barringer, Felicity. Census Bureau Places Population at 249.6 Million. *New York Times*, Dec. 27, 1990, p. A1.